ROBUST SENSITIVITY ANALYSIS FOR

MULTI-ATTRIBUTE DETERMINISTIC HIERARCHICAL VALUE MODELS

THESIS

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Abstract
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Subject Terms
Sensitivity Analysis, Multi-Attribute Additive Utility Models, Hierarchical Value Models, and Parametric Sensitivity Analysis

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Yucel R Kahraman

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Abstract

There is a wide array of multi-attribute decision analysis methods and associated sensitivity analysis procedures in the literature. However, there is no detailed discussion of sensitivity analysis methods solely relating to additive hierarchical value models. The currently available methodology in the literature is unsophisticated and can be hard to implement into complex models. The methodology proposed in this research builds mathematical foundations for a robust sensitivity analysis approach and extends the current methodology to a more powerful form. The new methodology is easy to implement into complex hierarchical value models and gives flexible and dynamic capabilities to decision makers during sensitivity analysis. The mathematical notation is provided in this study along with applied examples to demonstrate this methodology. Global and local sensitivity analysis are considered and implemented using the proposed robust technique. This research provides consistency and a common standard for the decision analysis community for sensitivity analysis of multi-attribute deterministic hierarchical value models.

ROBUST SENSITIVITY ANALYSIS FOR MULTI-ATTRIBUTE DETERMINISTIC VALUE MODELS

I. Introduction

Background

Value focused thinking is a decision making methodology proposed by Ralph Keeney in 1992. Keeney states, "Values are what we care about. As such, values should be the driving force for our decision making. They should be the basis for the time and effort we spend thinking about decisions." He also adds "But this is not the way it is. It is not even close to the way it is." (Keeney, 1998:3)

In our daily life, we often make our decisions by comparing the alternatives presented to us. Keeney terms this is alternative-focused thinking. But he emphasizes, "Values are more fundamental to a decision problem than alternatives." (Keeney, 1998:3) The values ultimately help to the decision maker to determine the relative desirability of consequences. Values, therefore, should be the basis for our decisions.

Value focus thinking (VFT) forces the decision makers to identify what they want (value) and builds the structure to achieve these goals (Keeney, 1998:4). It is a systematic and powerful approach, especially when facing decision situations where there are multiple, potentially competing objectives requiring considerations of trade offs among these objectives (Kirkwood, 1997:1).

Although VFT is a relatively new methodology (Keeney, 1992), it has been widely used in many areas of decision making. These include deciding new policies (Keeney, 1998: 342-371), selecting and implementing security procedures for transporting nuclear waste (Keeney, 1998: 295-307), and selecting construction sites for critical installations (Keeney and Raiffa, 1993: 436-472). VFT attempts to convert all the qualitative and subjective input data that is valued in a decision situation into objective and quantitative measures for alternative comparison. This structural quantification helps decision makers to evaluate the alternatives in term of their values and select the most valued one among these available alternatives. In addition, if there are unmet values, the VFT approach can help to develop new alternatives. Like other decision making disciplines, VFT also uses sensitivity analysis to determine how robust and sensitive their decision are in terms of the changes to the input variables.

Kirkwood directly discusses sensitivity analysis in value-based multi-objective models (Kirkwood, 1997:82-85). The methodology, however, is illustrated with a very basic, simple example (a one tier hierarchy). The outlined method can be misunderstood when implemented in complex models, particularly in cases where the value hierarchy is weighted locally on its sub-objectives.

Sensitivity analysis has not received a great deal of discussion in the literature relating to additive value-models. There are some proposed methods such as entropy-based and least squares procedure of Barron and Schmidt (1998), the flat maxima principle of Von Winterfeldt and Edward (1986) using multi-attribute utility theory or a Bayesian model, or linear programming like sensitivity analysis in decision theory of Evans (1984). Other academicians, like Rios Insua (1990), Samson (1998), have

evaluated the subject in broader context of decision analysis. However there is no specific method widely recognized in the literature encompassing additive value models.

Problem Statement

Sensitivity analysis is discussed extensively in multi-attribute decision making area, but there is no detailed methodology solely relating to additive value models. Because of this, the application of sensitivity analysis in value-based additive decision models is very limited and the exact implementations of approaches vary a great deal. This thesis expands the current sensitivity analysis methodology used for hierarchical value models and provides a common mathematical framework for complete sensitivity analysis. The framework uses a parametric approach for sensitivity analysis.

The first issue this thesis research addresses is demonstrated by Kirkwood (1997) and is also used in decision analysis software Logical Decisions. Kirkwood changes the desired attribute weight for sensitivity analysis from 0 to 1 while maintaining the proportional ratio between all other attribute weights constant. The list of ranked alternatives is examined for changes after the final scores are calculated over the entire sensitivity range for desired sensitivity attribute. In this study, this approach will be called global manipulation of the model weights because the method manipulates all the weights relating to a complete model at the same time. On surface, the method looks fine. However, if the desired attribute weight for sensitivity analysis is selected from a lower tier in a locally weighted value hierarchy, the new weight distribution does not reflect the model's exact intent. This results because the independent weights in different branches of a hierarchy are calculated in a dependent fashion. The applied method

causes no problem if the value hierarchy is weighted globally. The approach suggested in this thesis will be called robust manipulation. The details will be shown in following chapters. In addition to comparing global and local weight manipulation, this thesis provides the basic mathematical representation to handle either type of sensitivity analysis. Furthermore, the parametric approach taken allows more robust sensitivity analysis not currently found in the literature. There is no need to limit the manipulation of the attribute's weights to only a proportional approach. If the actual setting dictates, some of the weights may stay at their exact initial values when the others are changing. The implications of this approach will also be discussed in following chapters.

Problem Approach

This thesis first includes a literature review of issues concerning sensitivity analysis of multi-attribute value models. The current methodology is exercised and expanded with new recommended approaches. The mathematical foundation is provided to generalize sensitivity analysis using a parametric approach. The discussed approaches are demonstrated using the mathematical notation defined by this research. The results are shown and explained with their implications using an example problem. The advantages and disadvantages of current and proposed methods are discussed in the conclusion.

Research Scope

There are several different methods for conducting multi-attribute decision making and accompanying sensitivity analysis, such as weighted sum method, weighted

product method, and the Analytic Hierarchy Process (AHP). This research focuses on only weighted sum models and its applicable sensitivity analysis.

Furthermore, this research focuses on deterministic hierarchical value models. Uncertainty and risk analysis of a decision situation are not included as a part of sensitivity analysis study. It is also assumed that models are weighted by using simple numerical weighting techniques, like direct weighting or swing weighting, to show the immediate and active involvement of decision makers in multi-attribute decision analysis process.

Overall, the assumptions of this research are limited to deterministic, discrete, single decision maker, constant weight, and weighted sum hierarchical value-focused models.

Assumptions

The examples covered in this research will preserve all the assumptions of a multi-attribute value model that are covered by Kirkwood (1997:16-20) The models are complete, nonredundant, decomposable or independent, operable and, small size.

This thesis does not refer to any other phases of the multi-attribute decision making process except the part sensitivity analysis phase. The models are assumed complete and ready for this analysis.

Overview and Format

Chapter 2 covers the literature review pertinent to sensitivity analysis in multi-attribute decision models. Chapter 3 discusses the global versus local issue and expanded mathematical explanation of robust parametric sensitivity analysis. Chapter 4 uses a

value model example for the implementation of the methodologies shown in previous chapter. Chapter 5 concludes the results in a descriptive way and also provides recommendations and possible research areas for future analysis.

II. Literature Review

Introduction

This literature review is limited to the subjects and methods relating to weighted-sum models in multi-attribute decision making field. A more complete discussion of decision analysis and value focused thinking can be found in Keeney (1998), Kirkwood (1997), Keeney and Raiffa (1993) and Triantaphyllou (2000). This chapter gives brief explanations of subjects discussed in this research to build a foundation for proper implementation of sensitivity analysis in the following chapters.

Introduction to Decision Analysis

Everyone makes important decisions about their personal needs and problems, such as finding the right job, going to right school, deciding who and whom to marry, and so forth. In addition, managers in large companies, commanders in armed forces, and high-level government officials must constantly make important and complex decisions. These individuals are facing increasingly complex decision problems on daily basis. These decision makers are responsible for making good decisions; not an easy task. Decisions are hard because of their complexity and difficult because of the uncertainty inherent to them (Clemen, 1996:2-3). Decisions typically involve multiple objectives where slight changes in input variables may lead to totally different choices (Clemen, 1996:3). Education and experience are main inputs for a decision maker, but there are also some tools designed to ease the decision maker's job. The field of decision analysis offers many of these tools.

Decision analysis mainly deals with two repeated problems of decision making, uncertainty and multiple objectives (Edwards and Von Winterfeldt, 1986:2). Decision analysis "balances uncertainties and outcomes in accordance with the judgments and preferences of decision maker" (Edwards and Von Winterfeldt, 1986:5).

Decision analysis is a "prescriptive approach designed for normally intelligent people who want to think hard and systematically about some important real problems" (Keeney and Raiffa, 1993:xv)

Samson describes decision analysis as a useful technique in solving complex decision problems:

> Decision process and the analysis that can be done to support decisions revolve around three elements: problems, conceptual frameworks, and techniques. Decision analysis, which both conceptual framework and set of techniques that managers are finding useful in dealing with complex problems. (Samson, 1998:2)

Decision analysis attacks decision problems that heavily rely on probabilities, values, uncertainties, and, most importantly, judgments. Judgments are trade-offs. "They depend on decision makers assessment of the relative desirability of the available options on each dimension and on his or her feelings about the relative importance of these dimensions" (Edwards and Von Winterfeldt, 1986:6). Judgments are relevant for making good decisions. Judgments are not excluded from decision process like in analytical procedures of management science and operation research (Clemen, 1996:5).

Decision analysis does not take over the job of the decision makers. It neither guarantees high probabilities of success nor solves the uncertainty issues of complex decision problems. It can only help decision maker to understand the decision problem thoroughly. Properly executed, decision analysis will show clearly "the structure of

problems as well as the uncertainty, objectives and trade-offs inherent in the alternatives and outcomes and possibly recommend a course of action" (Clemen, 1996:3-4). This understanding will help the decision maker to make decisions with their eyes open (Clemen, 1996:4).

Multi-Objective Decision Analysis

A multi-criteria decision analysis problem has multiple objectives and requires consideration of tradeoffs between these objectives. There are many multi-objective decision methods available in the literature and they can be classified under different groups. They can be termed as deterministic, stochastic, or fuzzy multi-objective decision methods if they are classified according to the data they use (Triantaphyllou, 2000: 3). Multi-attribute decision analysis problems can be classified according to the number of decision makers included in the situation, either single or group. They can also be classified according to the salient features of the information they use. This final classification tree is shown in Figure 2-1, taken from Chen and Hwang.

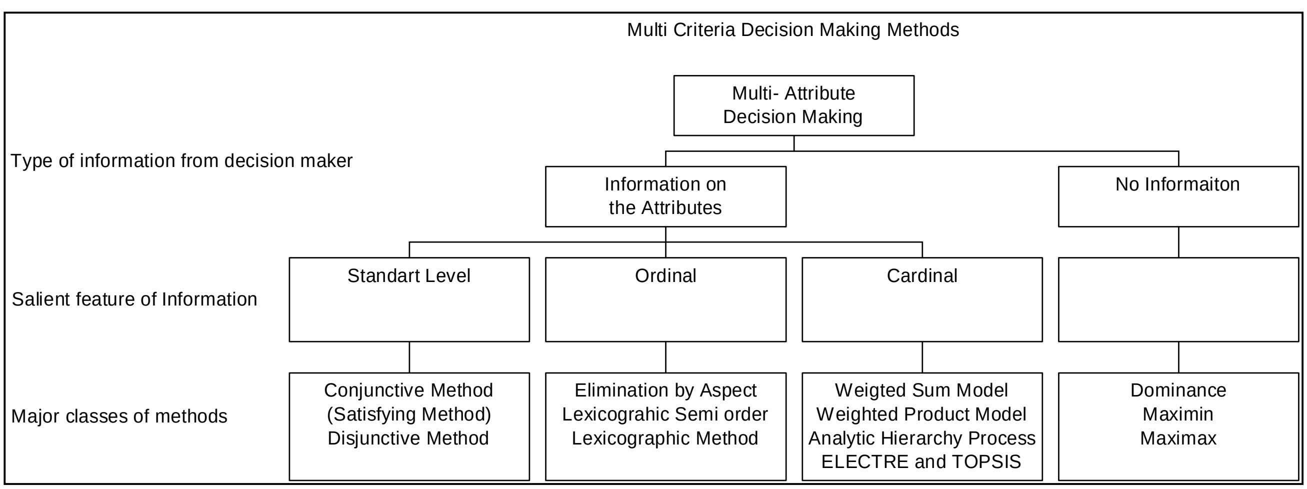

Figure 2-1. Taxonomy of MCDM Methods (Chen and Hwang, 1991)

Despite this vast variation in methods, all the approaches have something in common; they all assisting the decision maker in multi-attribute complex decision situations. This commonality is expressed by Insua (1999) as followed:

> One of the most interesting phenomena within the field of multi-criteria decision analysis (MCDA) is the various schools of thought. Despite their foundational and philosophical differences, at an abstract level, most multi-criteria methods essentially require subjective inputs from the decision maker (DM), which when combined with data relative to the alternatives and states of the problem, lead, through an algorithmic procedure, to a subset of good alternatives, perhaps one. (Insua, 1999: 117)

Insua points out important attributes of the multi-criteria decision analysis. He talks about the different groups with different backgrounds. He also emphasizes the subjectivity coming from decision maker that can have considerable effects on the outcome of analysis (Insua, 1999).

This research on sensitivity analysis will focus on deterministic, single decision maker, weighted sum models (hierarchical). Furthermore, value focused thinking is the desired approach in the building phase of hierarchical decision models.

Background on Value Focus Thinking and Multi-Objective Decision Analysis

When faced with decision problems with multiple competing objectives, like buying a car, or evaluating a new job opportunity, people tend to identify the alternatives and select the most suitable one according to their objectives and criteria evaluation. This is how people are raised and learn about solving these problems. Keeney explains this situation as followed:

> Decision making usually focuses on the choice among alternatives. Indeed, it is common to characterize a decision problem by the alternatives available. It seems as if the alternatives present themselves and decision problem begins when at least two alternatives have appeared. Descriptively, I think this represents almost all decision situations. Prescriptively, it should be possible to do much better. (Keeney, 1998:3)

Keeney calls this classic decision problem solving technique as alternative-focused thinking. First, available alternatives for decision problem are selected and second, they are evaluated according to the values and objectives of decision maker (Keeney, 1998:3). Keeney also termed these decision making situations as decision problems and says that every decision problem builds a decision opportunity to create alternatives based on our values. Keeney proposes another approach that he termed value-focused thinking (Keeney, 1998:8). He states, "values should be the driving force for our decision making" (Keeney, 1998:13).

Value-focused thinking is similar to the common alternative-focused methodology but it reverses two steps of classical method. Instead of comparing alternatives first, and then the values, the values and objectives are specified first and then related to the alternatives. This changes the focus of the decision making process from alternative driven to value driven.

Value focused thinking is resourceful, creative, and much broader than alternative focused thinking. It gives to decision maker the power of striving toward the best possible results relative to their values (Keeney, 1998:47-51). The value focused

thinking method consist of five dependant steps. Recognizing a decision problem, specifying values and objectives, creating alternatives, evaluating alternatives, and selecting an alternative from the evaluated set of alternatives (Keeney, 1998:49).

Value focused thinking converts a multi-objective decision problem with qualitative features into a quantitative, descriptive value hierarchy. Once this is completed, the alternatives are evaluated against this hierarchy. They are not compared directly to each other as in alternative focused thinking; rather, they are compared in terms of their final value scores. The alternatives are evaluated in terms of decision makers' preferences (values) about the decision situation. After all these steps are done, the final phase, sensitivity analysis, comes into play. The sensitivity analysis phase investigates how robust the results are in terms of changes in input variables. This research focused on this phase and assumes the other steps in the procedures are accurately completed.

There are other multi-attribute decision making models based on values. They are similar, depending on multi-attribute utility theory, but they have some different aspects of manipulating the data in deciding the utility functions and weights. One of them is SMART.

SMART was developed by Ward Edwards. Edwards built off of the multi-attribute utility theory and indifference methods proposed by Raiffa (1969). Edwards focused on the elicitation techniques in multi-attribute utility theory and ensuring that they were more simple and robust. He concentrated on numerical estimation techniques,

like direct rating and swing techniques in the estimation of single attribute value functions and weights for overall hierarchy instead of using cross-attribute indifference or cross-attribute strength of preferences methods. With its simplicity, SMART became popular and evolved into a strong approach in multi-attribute decision area (Von Winterfeldt, Edwards, 1986: 278-279).

Kirkwood (1997) also proposes a similar methodology to solve multi-attribute decision problems where decision makers face "multiple competing objectives that requires considerations of tradeoffs among these objectives" (Kirkwood, 1997:1). He references both methodologies proposed by Keeney and Edwards and combines them as a whole termed strategic decision making.

This research concentrates on multi-attribute additive value models that heavily depend on value focused approach proposed by Keeney. It is further assumed that the weightings of attributes are done using numerical estimation approaches proposed by Edwards in SMART and Kirkwood as swing weighting in Strategic Decision Making. The overall hierarchical models are an important part in this study.

Hierarchical structuring is a method that aids people in dealing with complexity. Complex decision problems can be easily structured in homogeneous clusters of factors using a hierarchical approach method (Forman and Gass, 1999: 470).

The weighting strategy, top to bottom or bottom to top approach, also plays an important role in the evaluation of multi-attribute deterministic hierarchical value models.

The bottom to top and top to the bottom weighting strategies are explained in Chapter 3 of this research. The possible methodologies to weight a hierarchy are not a part of this research and they are not evaluated.

The proposed sensitivity analysis method in this research is applicable to all additive value models regardless of the building and weighting strategy used. The decision makers can weight the hierarchy either locally or globally. Both techniques are valid in the evaluation of the alternatives against the hierarchy, but the user must use the appropriate technique associated with their weighting strategy (local or global) in the sensitivity analysis phase of the problem. The single dimensional value function elicitation, which is necessary in value focused thinking, is not considered in detail.

This research concentrates on hierarchical value models as described above because they capture all the details and goals pertinent to the decision situation according to the decision maker's preferences and values. In addition, alternatives are not required to build a value model. Multi-objective decision environments can be effectively structured without the presence of any alternative. This powerful characteristic of developing a model based on the decision makers' values, without any specific alternatives is the main reason for the selection of value focused thinking as the primary research focus of the study. The value hierarchies can stay intact as long as the decision maker preferences remain unchanged. New alternatives can be evaluated easily and these alternatives do not affect the established model.

Sensitivity Analysis and Multi-attribute Decision Analysis

The intention of sensitivity analysis is to judge how an outcome of a quantitative analysis depends on the inputs. Pannell explains sensitivity analysis as "the investigation of potential changes and errors and their impacts on conclusions to be drawn from the model" (Pannell, 1997:139). Samson states "a sensitivity analysis generally involves checking the effects of the model assumptions on the model solution." (Samson, 1988:269)

Decisions are made according to the model output driven by the input data. Decision makers are interested in knowing how much the decision is affected if the inputs about the decision situation have changed. Insua (1999) directly considers sensitivity analysis issue in multi-criteria decision analysis. He also briefly touches the importance of sensitivity analysis in other decision fields like operations research, and statistics. He points out the importance and difference of sensitivity analysis (SA) in multi-criteria decision making procedure:

> Traditional reasons (limited analysis time or computational resources, imprecision in beliefs and preferences, ill-defined data, ...) there contained to justify sensitivity analysis apply in multi-criteria decision analysis. However, in our field, SA is perhaps more important, since it may be the means of explaining to the decision maker the implications and possible inconsistencies of his judgments. (Insua, 1999: 117)

Fiacco (1983) explains the importance and usefulness of sensitivity analysis as follows:

> A methodology for conducting a (sensitivity) analysis ... is a well established requirement of any scientific discipline. A sensitivity and stability analysis should be an integral part of any solution methodology. The status of a solution cannot be understood without such information. This has been well recognized since the inception of scientific inquiry and

has been explicitly addressed from the beginning of mathematics. (Fiacco, 1983: 3)

Clearly, sensitivity analysis is a critical step in any decision analysis. This research looks specifically at sensitivity analysis in deterministic, multi-attribute, hierarchical, and single decision maker value models.

The different uses of sensitivity analysis, from Pannel, are summarized in the Table 2-1.

Table 2-1. Uses of sensitivity analysis (Pannell, 1997:140)

1.	Decision Making or Development of Recommendations for Decision Makers
1.1	Testing the robustness of an optimal solution.
1.2	Identifying critical values, thresholds or break-even values where the optimal strategy changes.
1.3	Identifying sensitive or important variables.
1.4	Investing sub-optimal solutions.
1.5	Developing flexible recommendations which depend on circumstances.
1.6	Comparing the values of simple and complex decision strategies.
1.7	Assessing the "riskiness" of a strategy or scenario.
2.	**Communication**
2.1	Making recommendations more credible, understandable, compelling or persuasive.
2.2	Allowing decision makers to select assumptions.
2.3	Conveying lack of commitment to any single strategy.
3.	**Increased Understanding of Quantification of the System**
3.1	Estimating relationships between input and output variables.
3.2	Understanding relationships between input and output variables.
3.3	Developing hypotheses for testing.
4.	**Model Development**
4.1	Testing the model for validity or accuracy.
4.2	Searching for errors in the model.
4.3	Simplifying the model.
4.4	Calibrating the model.
4.5	Coping with poor or missing data.
4.6	Prioritizing acquisition of information.

Triantaphyllou summarizes the current state of sensitivity analysis issues as followed:

> There is considerable research on sensitivity analysis for some operations research and management science models such as linear programming and investment analysis. For example, in a sensitivity analysis approach for linear programming, Wendel (1992) utilized a tolerance approach to handle variations in the parameters of more than one term (in the LP sense) at a time. Furthermore, that type of sensitivity analysis is considered a post-optimality step. That is, the analysis is done after the optimal decision is determined. However, research on sensitivity analysis in deterministic multi-criteria decision making models is limited. (Triantaphyllou, 1997:151)

Von Winterfeldt and Edwards (1986) research sensitivity analysis in expected value and multi-attribute utility models. They build their cases upon models structured under uncertainty. They point out the similarities between two types of models and evaluate them under the Flat Maxima Principle. The Flat Maxima Principle assumes that empirical observations of decision making problems are robust to reasonable variations in the parameters of the problem. Von Winterfeldt and Edwards also discuss dominance, stating that dominance, if it exists, will totally eliminate the need for sensitivity analysis in a decision model (Von Winterfeldt and Edwards, 1986:387-447).

Rios Insua (1990) focuses on sensitivity analysis in multi-objective decision making, researching it in great detail. His work mainly finds its foundation on Bayesian decision analysis and decision models constructed under uncertainty. He demonstrates that the Flat Maxima Principle proposed by Winterfeldt and Edwards is not always true.

Samson (1988) brings an interesting concept to the discussion of sensitivity analysis in decision analysis. He identifies the sensitivity analysis procedure not as a post optimality analysis like other decision making disciplines, but as integrated part into

every step of a decision making process. He explains the classic and proposed methodologies with tables and then gives broad explanations about the implementation of recommended strategy into every step of decision making. Figures 2-2 and 2-3, used by Samson, illustrate his approach.

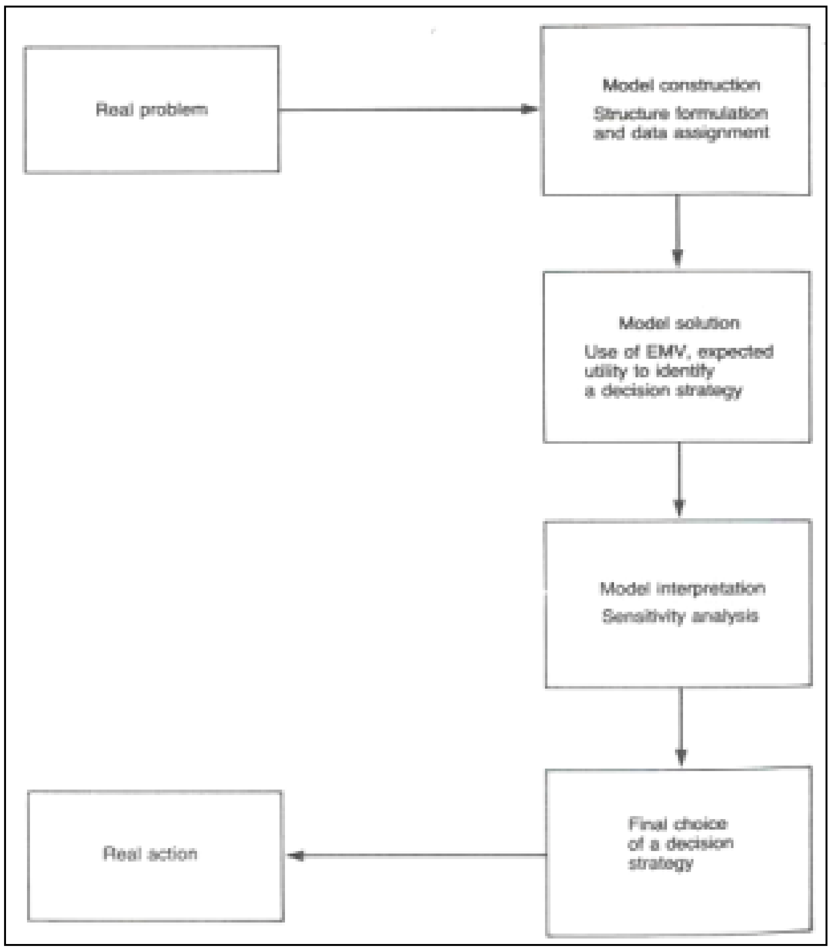

Figure 2-2. The Basic Decision Analysis Process (Samson, 1988:270)

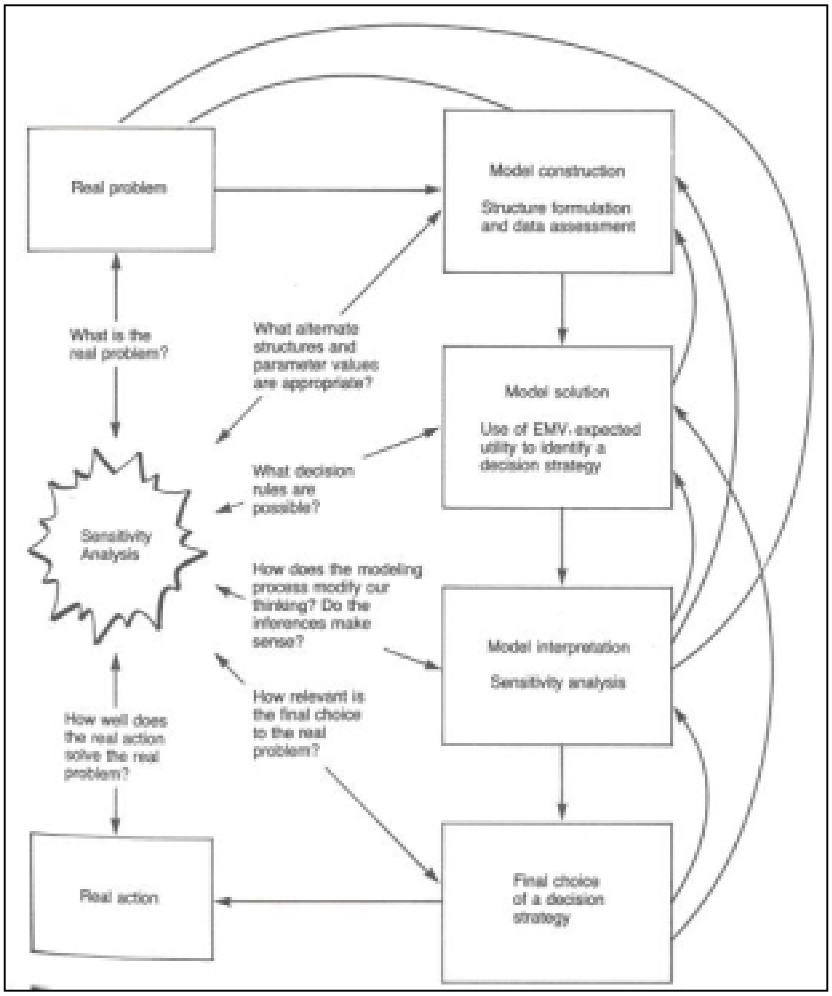

Figure 2-3. A Decision Analysis Process that Embeds Sensitivity Analysis Throughout all Steps (Samson, 1988:271)

His general approach is powerful and can give a resourceful insight to analysts. Making it applicable to every model requires a great deal of experience and interaction with different level decision makers.

Sensitivity Analysis Approaches in the Literature for Deterministic Additive Value Models

Keeney and Raiffa Recommendation

Keeney and Raiffa lay the foundation for sensitivity analysis in hierarchical value models. They mention that sensitivity analysis can be made either on single dimensional value functions (SDVF) or with the manipulation of the weights of the model. Sensitivity analysis on the SDVFs can be implemented to see how the alternative scores change if the value functions change. Manipulation of the model weights looks at how the alternative scores can change if the weighting of the value hierarchy changes. They briefly touch on these subjects but do not show any detailed illustration of the proposed methods. Specifically, they mention:

> By specifying a group of alternatives differing slightly in some feature, we can conduct a sensitivity analysis of the probabilistic inputs. Also, we can conduct sensitivity analysis of the preference structure by varying such parameters as the scaling constants in the multi-attribute utility function. In this way, different utility functions of members of a decision making group can be used to evaluate and rank the alternatives. This might clarify differences of opinion and suggest certain creative compromises if needed. (Keeney and Raiffa, 1993:352)

Kirkwood Method

Kirkwood, in his text, extends Keeney and Raiffa's discussion and applies it to value models. He prefers to conduct the sensitivity analysis on the weights of the attributes. He changes the weight of selected sensitivity attribute from 0 to 1 while keeping the other attribute weights proportionally. Equation 2.1 shows how Kirkwood is handling the calculation at a global area.

$$w_i = (1 - w_s) \cdot \left(\frac{w_i^0}{\sum_{i=1}^{m} w_i^0} \right) \tag{2.1}$$

In Equation 2.1 w_i represents all weights changing according to the sensitivity analysis, w_s represents the weight being analyzed and w_i^0 represents all the changing weights' original status in the first model. The ratio of the changing weights remains constant throughout the analysis. The scores of the alternatives are compared and the results are displayed on a two dimensional chart. Kirkwood focuses on "break even" or "crossover" points where he suggested selection would change from one alternative to another one. Keeping other weights proportionally is logical if the decision maker used a swing weighting or some pair wise comparison method at the global area. The weights at this area are compared with each other and posses a ratio derived from this comparison. This mentioned area concept is explained in Chapter 3. It is assumed in this research that the weighting is "global" if the hierarchy is expanded into its final tier and weighted at this area. Kirkwood used only one simple example to illustrate the procedure and did not show sensitivity analysis implemented for a complex hierarchy. Kirkwood's approach could cause the decision makers to drift away from his decision preferences about the weights if they perform sensitivity analysis on a locally weighted value hierarchy. The weight assumptions of the hierarchy will not stay the same as they are established in the original model. This potential short fall will be discussed in greater depth in the coming chapter.

Triantaphyllou Method

Triantaphyllou's method is applicable to three major multi-criteria decision methods. These methods are: the weighted sum model (WSM), the weighted product model (WPM), and analytic hierarchy process (AHP). His method attempts to identify the most important criteria weight and the most important criterion value score within the model. Triantaphyllou summarizes his approach:

> The decision maker can make better decisions if he/she can determine how critical each criterion is. In other words, how sensitive the actual ranking of the alternatives is to changes in the current weights of the decision criteria. Thus, in this chapter we examine two closely related sensitivity analysis problems. In the first problem we determine how critical each criterion is, by performing a sensitivity analysis on the weights of the criteria. This sensitivity analysis approach determines what is the smallest change in the current weights of criteria, which can alter the existing ranking of alternatives. In second problem, we use a similar concept to determine how critical the various performance measures of the alternatives (in terms of single decision criterion at a time) are in the ranking of the alternatives. (Triantaphyllou, 2000:132)

His method does not evaluate the final value scores of alternatives and does not check the relation between the weights of the complete model during the calculations. His calculations of most important performance measures requires too many combinations to evaluate and makes it difficult to effectively evaluate for decision makers. A more detailed explanation of his method can be found in Triantaphyllou (2000). His findings are general and give an overview of sensitivity analysis in multi-attribute decision models. He explains his conclusions taken out from his research as follows:

> The empirical contributions are related to the sensitivity analysis of changes in the weights of the decision criteria. We did not cover changes in the a_{ij} values with an empirical study because that would result in too many sensitivity scenarios under consideration for a given problem and thus divert the attention from central ideas. Recall that for a problem with *m* alternatives and *n* criteria there are mxn different a_{ij} values.

> The two most important empirical conclusions of this study are: (i) The choice of the MCDM (multi-criteria decision making) method or number of alternatives has little influence on the sensitivity results; and (ii) most frequently the most sensitive decision criterion is the one with the highest weight, if weight changes are measured in relative terms (i.e., as percentages), and it is the one with the lowest weight if changes are measured in absolute terms.
>
> The main observation of computational experiments is that the MCDM methods studied here, perform in similar patterns. These patterns refer to the frequency the criterion is with the highest (or lowest) weight is also the most critical criterion, when changes are measured in relative (or absolute) terms. Moreover, the same results seem to indicate that the number of decision criteria is more important than the number of alternatives in a test problem. (Triantaphyllou, 2000:165)

Software Packages Designed for Multi-Attribute Decision Analysis Problems

There are many software packages designed to solve the multi-criteria decision analysis problems. These software packages were reviewed to determine what type of sensitivity analysis tools they have and how they apply the sensitivity analysis. The information represented on Table 2-2 was solicited directly from the developers of the software packages by consultation on the phone.

Table 2-2. Software for Multi-Attribute Decision Analysis

Software Name	Vendor	Sensitivity Analysis	Local/Global
Decision Explorer	Banxia Software Ltd.	No	-
Team Expert Choice	Expert Choice, Inc.	Yes	Local
EXSYS Corvid	EXSYS, Inc.	No. Programable	-
ELECTRE 3-4	LAMSADE Softwares	Yes	Global
ELECTRE IS	LAMSADE Softwares	Yes	Global
ELECTRE TRI	LAMSADE Softwares	Yes	Global
Logical Decisions for Windows	Logical Decisions	Yes	Global
Netica	Norsys Software Corp.	No. Programable	-
DATA 3.5	TreeAge Software, Inc.	No. Programable	-
DATA Interactive	TreeAge Software, Inc.	No. Programable	-

It should be noted that not all the approaches are based on global weighting, although this fact is not always clearly stated. Where sensitivity analysis is programmable, it will be shown that is critical that the global versus local weighting distinction is clarified.

Conclusion and Direction

This chapter reviews the literature applicable for sensitivity analysis issues in multi-attribute deterministic value models. The chapter starts with a brief explanation of decision analysis and then builds the subject systematically into multi-attribute value models and ends with sensitivity analysis of deterministic value models. It is shown that the input data for a decision model can vary from deterministic, to stochastic and fuzzy sets. It is also summarizes the research focused on deterministic, single decision maker hierarchical value models. The discussion on current applicable methods builds an understanding for the remaining chapters.

In the following chapter, the global and local manipulation of weights is discussed. The new parametric robust sensitivity analysis, capable of global and local parametric sensitivity analysis, is explained with proper mathematical notation. Chapter 4 illustrates this new methodology with an example hierarchy. This thesis ends with summary remarks and suggestions for future research in Chapter 5.

III. Methodology

Introduction

This chapter begins with a discussion of the "global" versus "local" manipulation issue of the weights in a decision analysis sensitivity analysis. Next, a review of current sensitivity analysis methodology is presented. Shortcomings of the current methodology are then discussed. Finally, a robust mathematical sensitivity analysis methodology is presented to handle both types (local and global) of sensitivity analysis for the weighted hierarchical sum models. The proposed methodology is designed to handle the sensitivity analysis issues for hierarchical value models, however, it is applicable to all weighted sum models for global sensitivity analysis regardless of methods used in their construction phase, hierarchical or not hierarchical. The proposed methodology classifies the weights relating to a model in different categories and then manipulates them according to this classification. The application of the methodology is explained on an example value hierarchy in Chapter 4.

Global versus Local Sensitivity Analysis in a Value Hierarchy

The terms relating to value hierarchies are defined to ensure clarity of the issues presented in this chapter. Value hierarchies consist of tiers and branches. The hierarchies also have local and global areas where the sensitivity analysis can be exercised. The sensitivity areas and associated weights are the fundamental elements of the proposed methodology; the terms are explained on an example hierarchy. In value focus thinking, hierarchies are detailed representations of decision situations. They show the main objective, the sub-objectives and the supporting sub-objectives until the point

where the objectives cannot be divided further into any sub-objectives. At this point single dimensional measures are created. All parts of hierarchy are mutually exclusive and collectively exhaustive. Tiers can be defined as the layers of the hierarchical value structure. The final layer, where a value cannot be expanded further, creates the last tier.

Figure 3-1 shows the tiers of an arbitrary value hierarchy, containing two tiers. The first tier represents three sub-objectives stemming from the main objective. The first and third sub-objectives have further sub-objectives building the second tier of the hierarchy. If a sub-objective (sub-objective 2) does not expand like its peers, a placeholder for this unexpanded node is moved into the next tier to maintain the completeness of the hierarchy in the lower areas.

The placeholder does not impact the structure of the hierarchy but it aids the sensitivity analysis calculations. The placeholder concept is important for the correctness of sensitivity analysis when the analysis is conducted in the tier containing the expanded placeholder. This is shown on Figure 3-1. The final tier captures the overall measures of the hierarchy.

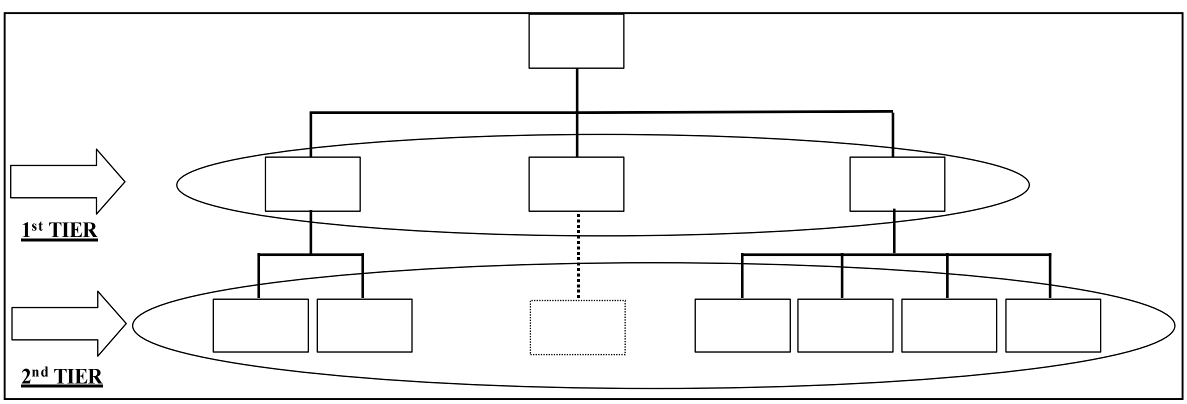

Figure 3-1. The tiers of a value hierarchy

The sub hierarchies attached to the main objective can be named as branches of value hierarchy that are shown on Figure 3-2. The branches represent the independent and collectively exhaustive objectives of a value hierarchy. Further down in the hierarchy, branches can be identified as sets of values or measures that extends from a common node.

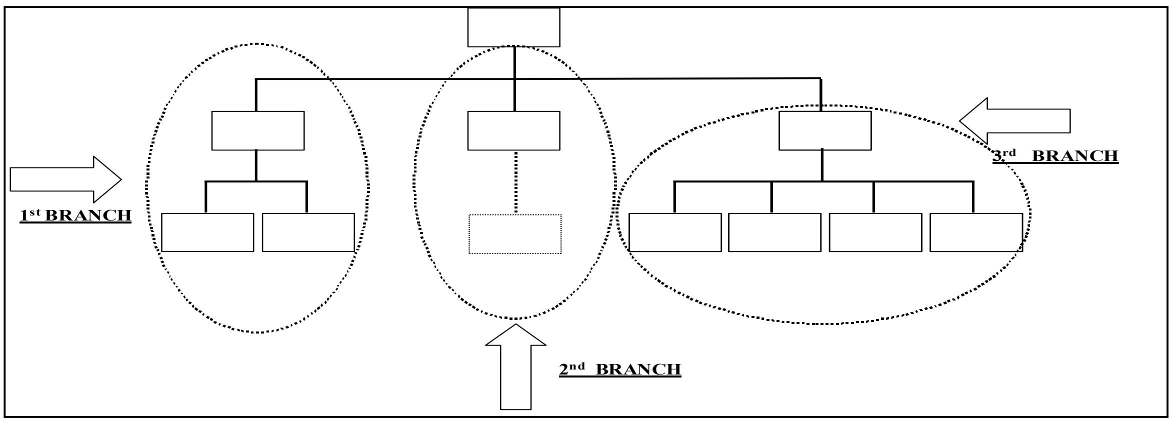

Figure 3-2. The branches of a value hierarchy

The global and local manipulation of the weights in sensitivity analysis of value hierarchies is proposed in this research. The areas for these two types of sensitivity analysis are shown on an example value hierarchy in Figure 3-3 and 3-4. The area of sensitivity analysis is defined as global (across branches) in this research if the sensitivity analysis is conducted across an entire tier on the value hierarchy. Furthermore, when doing global sensitivity analysis, the global weights are manipulated. This and the calculations of global and local weights will be reiterated later in this chapter. As explained before, the tiers represent the entire model in different detail stages. Local sensitivity analysis is exercised on a tier within a branch representing a sub-objective. Local areas are detailed representations of the objective (node) above them (within the

branch). They present a portion of the entire model relating to their objective parent. Local weights are used during this local sensitivity analysis. The sensitivity analysis can be done globally or locally anywhere in the value hierarchy.

It is important to highlight some key characteristics of value hierarchies. The first tier weights are local and at the same time global. The final tier is the most detailed representation of the value hierarchy. Generally, all weighted sum models are represented at this area, even if they were not built hierarchically.

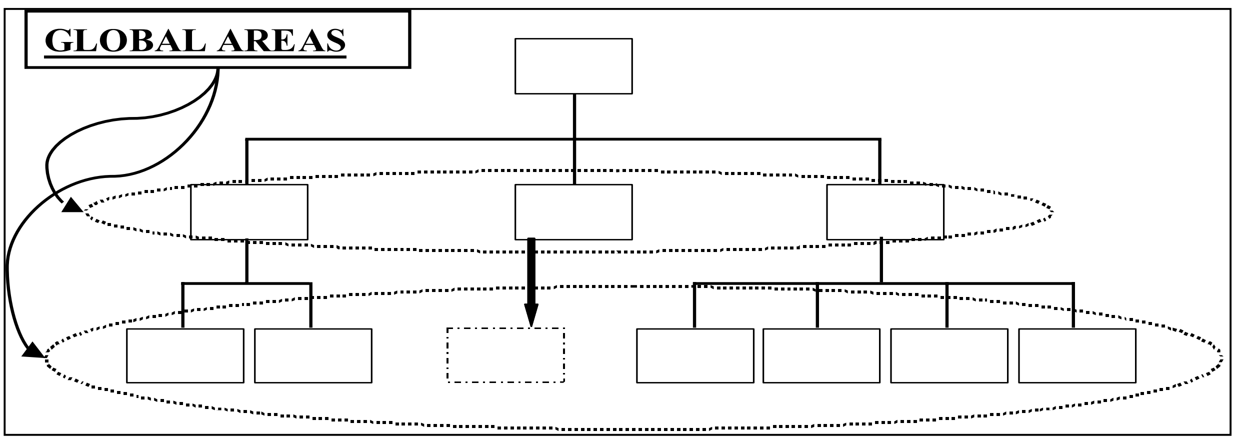

Figure 3-3. The global areas for sensitivity analysis

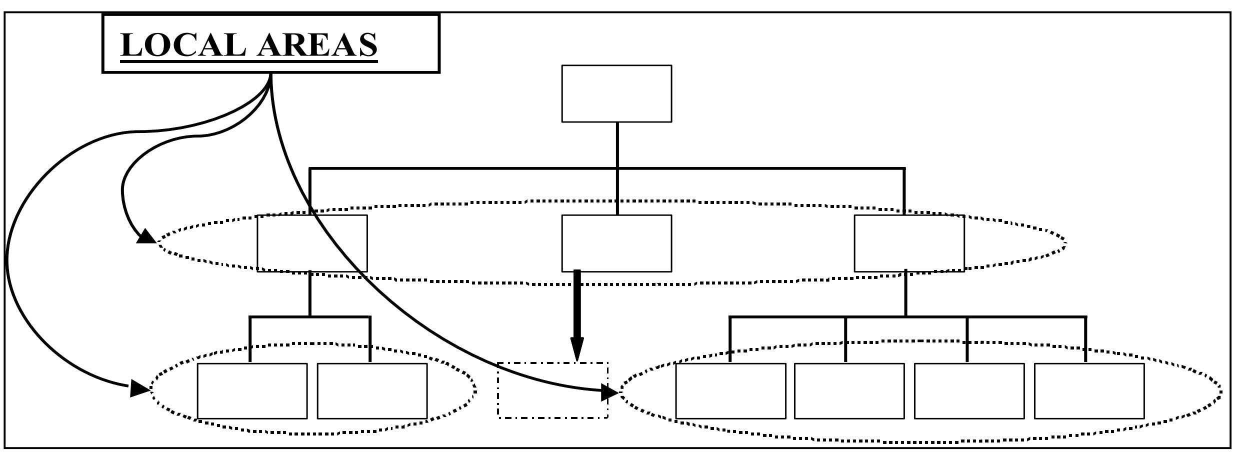

Figure 3-4. The local areas for sensitivity analysis

This research suggests the sensitivity analysis be done relative to the weighting methodology used in the weighting of the hierarchy. If the hierarchy is weighted globally, then the sensitivity analysis should be done globally. Similar condition holds for using local sensitivity analysis with local weighting. Otherwise, the independence assumption used in local weighting of the hierarchy will perish. Global sensitivity analysis conducted on an entire tier manipulates all the weights at once, even if there is no relation between the global weights in different branches. Therefore the correct sensitivity analysis should be conducted based on the weighting methodology applied. The mentioned weighting strategies (global and local) are discussed in the following section.

Weighting Strategy

The method used to weight the hierarchy is extremely important when conducting sensitivity analysis. During the global weighting process, all the measures are simultaneously compared. After the hierarchy is weighted globally at the final tier, local weights for every node can be calculated by dividing the global weight of the node with the total of the global weights in the local area. This strategy is a bottom to top approach.

A second approach is to weight the hierarchy beginning at the top tier and locally within each branch at each tier by using the simplifying feature of the hierarchy. The global weights are calculated by multiplying the local weight of a node by the local weights of those nodes above them in the hierarchy within the same branch. This is considered a top to bottom approach.

This thesis proposes a sensitivity analysis methodology that is based on the weighting strategies used during the weighting process of hierarchy. Local and global sensitivity analysis can be exercised using the new mathematical approach proposed.

Current Methodology

Current sensitivity analysis methodology for additive weighted sum models can be exercised either on hierarchy weights or on single dimensional value functions as it is explained in Chapter 2. As stated in Chapter 2, sensitivity analyses on the single dimensional functions are often not practical. Therefore sensitivity analysis is most commonly exercised on model weights.

Current methodology varies the weight of selected sensitivity measure from 0 to 1 while keeping the other attribute weights proportionally. As discussed, such an analysis uses Equation 2.1 while making the computations of the sensitivity weight. However, there are some areas to clarify to implement this methodology into complex hierarchical value models that are referred to as shortcomings of the current methodology in this thesis.

Shortcomings of the Current Methodology

The methodology for sensitivity analysis of weights currently in use is not explained in detail in the available literature. This is particularly true if one wishes to implement it into complex hierarchical value models. Mainly, it does not clarify if the weights should be manipulated locally or globally during the calculations. Furthermore, during the calculations of the other weights, current methodology keeps the original proportionality between the weights of the model. That may not be accurate in some

cases. The decision makers could be sure the states of some measures' weights during computations, not wishing for them all to change. The current methodology also does not allow calculating the other weights in the analysis parametrically; with a preference change a reallocation of weights might be to only some of the other measures, not all.

If Equation 2.1 is used for sensitivity analysis on a locally weighted value hierarchy, the weight calculations would not represent the decision maker's exact preferences about the attributes. As explained previously, this method violates the independence assumption used by local weighting strategy. In the local weighting strategy the weights in different branches are assigned independently. The decision maker assigns the weights in local areas. All measures are not considered at once in the weighting process. Only local sub-objectives or measures are evaluated while weighting the hierarchy.

The methodology proposed by this research eliminates this problem and gives the decision maker the ability to exercise local or global sensitivity analysis regardless of the weighting method used. The decision maker can select the appropriate sensitivity analysis according to the strategy used in the weighting process of the value hierarchy. The new proposed methodology also gives to the decision makers the power to manipulate the weights according to their preference. Decision makers can hold their preferences as they were in original hierarchy (dependent weights are proportional) or can change the weights according to the new preferences arising in the analysis phase (dependent weights are parametric).

Proposed Robust Sensitivity Analysis

The proposed robust sensitivity analysis on the weights of weighted sum models is outlined in a six-step process. The proposed sensitivity analysis methodology covers the current methodology used in the field, eliminates the global or local manipulation of weights problem and provides additional power by allowing the decision makers to manipulate the weights according to their latest preferences about the decision situation.

There are two important factors to consider when conducting sensitivity analysis for hierarchical value models. Will the sensitivity analysis be conducted globally or locally? And will the sensitivity analysis be implemented using a proportionality or parametric approach? Both of these factors are considered in the methodology provided.

Step 1: Decide the Sensitivity Area for Analysis

The manipulated weights are local or global weights according to the selected area for sensitivity analysis. After the weight for sensitivity analysis is decided, the area for sensitivity analysis should also be decided to conduct the sensitivity analysis (local or global). The sensitivity area is local if the hierarchy is weighted locally with a top to bottom approach. The original manipulated weights should be local. The sensitivity area is global if the hierarchy is weighted globally with a bottom to top approach and the original manipulated weights should be global.

Step 2: Define the Sets for the Analysis

Defining the sets in the analysis is the second step in the proposed methodology. This step helps decision makers to conduct the analysis using a proportionality or

parametric approach. It also checks the sensitivity area for a second time. There are four sets to be defined to continue the analysis. These sets are:

N	=	The set of all weights in the area selected for sensitivity analysis (global or local)
S	=	The set of weights being considered during sensitivity analysis
I	=	The set of weights changing during sensitivity analysis
U	=	The set of weights unchanging during sensitivity analysis

This research focuses only on one-way sensitivity analysis; therefore, the set S only includes one weight. The other defined variables associated with the analysis are as follows:

n	=	The total number of weights in the area selected for sensitivity analysis (global or local)
p	=	The number of weights being considered during sensitivity analysis
r	=	The number of weights changing during sensitivity analysis
t	=	The number of weights unchanging during sensitivity analysis
n	=	p + r + t, this implies $\lvert N\rvert = \lvert S\rvert + \lvert I\rvert + \lvert U\rvert$

For example, suppose the selected area of the value hierarchy for sensitivity analysis has seven weights. The selected area for sensitivity analysis can be local or global depending on the preference of the decision maker about the sensitivity weight (the weight in the selected area for sensitivity analysis). Set N will include all the weights present in the selected area. Therefore, in this example, the set N can be defined as follows:

$$N = \{w_1, w_2, w_3, w_4, w_5, w_6, w_7\}$$

$$n = \lvert N\rvert = 7$$

For illustrative purposes, assume that w_4 is the sensitivity weight, the weights w_3, w_5, w_6, w_7 are changing weights, and w_1,w_2 are unchanging weights. Therefore the other defined sets would be look as follows:

$$S = \{w_4\}$$

$$p = |S| = 1$$

$$I = \{w_3, w_5, w_6, w_7\}$$

$$r = |I| = 4$$

$$U = \{w_1, w_2\}$$

$$t = |U| = 2$$

$$n = p + r + t = 7$$

<u>Step 3: Calculate the Parameters (α_i, bound for Δx)</u>

The decision makers continues with the calculation of parameter α_i and the bound for Δx. The parameter α is defined as the weight coefficient of elasticity. The weight coefficient of elasticity expresses the relative trade-off of hierarchy weights in relation to given changes in the weight(s) being analyzed during sensitivity analysis. The weight coefficient of elasticity allocates the distribution of the weight(s) being analyzed to the other hierarchy weights during sensitivity analysis. The value of α_s (weight coefficient of elasticity for sensitivity weight) is defined to be one. All α_u are zero allowing for the values of some weights to be held constant while varying others according to decision maker's trade-off. The α_i parameter is calculated according to the decision maker preferences (which maybe proportionality or parametric). The weight

coefficients of elasticity for proportionality case are calculated using Equation 3.1. If the decision maker decides to make a parametric analysis, the decision maker decides the weight coefficients of elasticity. This is analogous to setting the change vector coefficients in parametric programming in linear programming.

$$\alpha_i = \frac{w_i^0}{\sum_{i \in I} w_i^0} \tag{3.1}$$

$$\alpha_i = user \quad defined \quad for \quad paremetric \quad case$$

The Δx parameter represents the amount of change implemented to the set of weights according to their associated weight coefficient of elasticity. However, this change cannot be uncontrolled. The change on sensitivity weight should be bounded; otherwise it will destroy the assumptions relating to the weights in the hierarchy. These assumptions are that the weights are positive values and they sum up to 1 with in the area selected for sensitivity analysis. The parameter Δx can be either positive, showing an increase in the relative importance, or negative, showing a decrease in the relative importance. The bounds for variable Δx are defined as the largest change amounts on sensitivity weight in a negative and positive direction. After the bounds are calculated, the decision maker can decide the step size for score calculations within the bound by dividing the bounded area by the desired number of steps or he can divide by the step size desired (fidelity) to get the required steps. The bound for variable Δx can be calculated using the following inequality for the parametric case:

$$-w_s^0 \le \Delta x \le \min \frac{w_i^0}{\alpha_i}, \forall i \in I \tag{3.2}$$

Equation 3.2 presented above is valid for the proportionality case also. However, the bound for the change on the sensitivity weight can also be set by Equation 3.3 shown below. This is because all the changing weights will reach zero at the same time in the proportional case according to desired number of steps for the analysis:

$$-w_s^0 \leq \Delta x \leq \sum_{i \in I} w_i^0 \tag{3.3}$$

Step 4: Calculate the New Weights According to the Set Parameters

In this step, the decision maker calculates the new weights according to the set parameters for sensitivity analysis. The new weights (w_s, w_i, w_u) are calculated with Equations 3.4, 3.5 and 3.6 respectively.

$$w_s = w_s^0 + \alpha_s \Delta x \qquad s \in S \tag{3.4}$$

$$w_i = w_i^0 - \alpha_i \Delta x \qquad i \in I \tag{3.5}$$

$$w_u = w_u^0 - \alpha_u \Delta x \qquad u \in U \tag{3.6}$$

The new weight distribution must also satisfy the condition given by Equation 3.7.

$$\sum\left(w_s^0 + \alpha_s \Delta x\right) + \sum\left(w_i^0 - \alpha_i \Delta x\right) + \sum\left(w_u^0 - \alpha_u \Delta x\right) = 1 \tag{3.7}$$

Equation 3.7 can be represented with Equation 3.8 using Equations 3.4 through 3.6.

$$\sum w_s + \sum w_i + \sum w_u = 1 \tag{3.8}$$

Referring to Equations 3.4 through 3.7, the original weights are defined as follows:

w_s^0 = The original value of the weight undergoing sensitivity analysis
w_i^0 = The original value of the dependant (changing) weights for sensitivity analysis
w_u^0 = The original value of the unchanging weights for sensitivity analysis

The original weights defined above are either global weights if a global sensitivity analysis is exercised or local weights if a local sensitivity analysis is exercised.

Step 5: Calculate the Scores for New Weight Distribution

In this step, the final scores are calculated to show the new ranking between alternatives. Equation 3.9 is used to calculate the final scores for alternatives and to determine the ranking between the alternatives. In Equation 3.9 the indices *j* represents the total number of measures used to evaluate the alternatives. The indices *i* represent the total number of alternatives evaluated in the study. The variable a_{ij} shows the single dimensional attribute value score of i^{th} alternative on j^{th} measure. The variable w_j shows the j^{th} attribute's global weight. The scores are value and weight dependent. This study researches the effects of the changing weights during sensitivity analysis on the alternative scores. Therefore, the weighting strategy and sensitivity method are critical to reach conclusions in terms of exact preferences of the decision maker.

$$Score_i = \sum_{j=1}^{n} a_{ij} w_j \tag{3.9}$$

(Triantaphyllou, 2000:6)

Step 6: Show the Results on a Breakeven Chart

After all these calculations are done, the results can be presented on a two dimensional breakeven chart to see the effects of weight change on the selected weight for sensitivity analysis. The chart presented in Figure 3.5 is an example chart that will be presented later in global proportionality case example in Chapter 4.

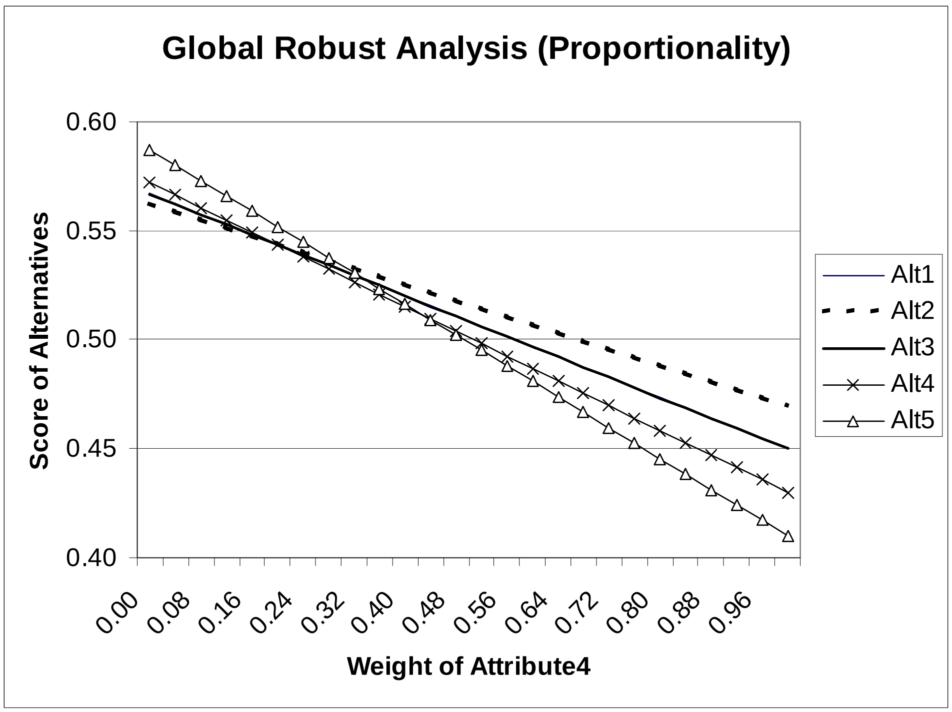

Figure 3-5. An example breakeven chart

Breakeven charts show the alternative scores across the set bound for sensitivity weight. They, therefore, allow the decision makers to see the possible ranking changes between alternatives within this set bound. This helps the decision makers to grasp the effects of the changes in terms of the final scores.

Global Robust Sensitivity Analysis

Global robust sensitivity analysis gives the decision maker the ability to conduct a sensitivity analysis at a global area of a value hierarchy either parametrically assigning new preferences or keeping the original proportionality. In this section, the current methodology, found in the literature (Kirkwood, 1997:82-85), is shown to be a special case of new proposed methodology (proportionality).

If the sensitivity analysis is done according to the current methodology (Kirkwood, 1997:82-85), U = {ø}. The number of changing weights (r) is equal to the number of all weights (n) minus the number of weights undergoing sensitivity analysis (p). The value of p = 1 is used throughout this research, matching the general literature.

Using the current methodology, global sensitivity analysis using proportionality is a special case of the new proposed methodology:

Equation 3.10 is currently used for sensitivity analysis in the additive value models.

$$w_i = (1 - w_s) \times \left(\frac{w_i^0}{\sum_{i \in I} w_i^0} \right) \tag{3.10}$$

(Kirkwood, 1997:82-83)

Substituting the Equation 3.4 for w_s into Equation 3.10, the Equation 3.11 is formed

$$w_i = \left[1 - \left(w_s^0 + \alpha_s \Delta x \right) \right] \times \left[\frac{w_i^0}{\sum_{i \in I} w_i^0} \right] \tag{3.11}$$

It is also known $\alpha_s = 1$ and $w_s^0 = 1 - \sum w_i^0$. If these values are substituted into Equation 3.11, Equation 3.12 is formed

$$w_i = \left[1 - \left(1 - \sum_{i \in I} w_i^0 + \Delta x\right)\right] \times \left[\frac{w_i^0}{\sum_{i \in I} w_i^0}\right] \tag{3.12}$$

by simplifying Equation 3.12, Equation 3.13 is formed

$$w_i = \left(\sum_{i \in I} w_i^0 - \Delta x\right) \times \frac{w_i^0}{\sum_{i \in I} w_i^0} \tag{3.13}$$

by removing the parenthesis from Equation 3.13, Equation 3.14 is formed

$$w_i = \sum_{i \in I} w_i^0 \frac{w_i^0}{\sum_{i \in I} w_i^0} - \Delta x \frac{w_i^0}{\sum_{i \in I} w_i^0} \tag{3.14}$$

Simplifying Equation 3.14 yields the Equation 3.15

$$w_i = w_i^0 - \Delta x \frac{w_i^0}{\sum_{i \in I} w_i^0} \tag{3.15}$$

It was previously stated that the weight coefficient of elasticity is calculated using Equation 3.1 in the proportionality case. If this is substituted for α_i into Equation 3.15, the Equation 3.16 is formed.

$$w_i = w_i^0 - \Delta x \, \alpha_i \tag{3.16}$$

Therefore the two methodologies are equivalent for standard application and, further, the current methodology is a special case of the proposed methodology.

$$w_i = (1 - w_s) \times \left(\frac{w_i^0}{\sum_{i \in I} w_i^0} \right) \equiv w_i^0 - \Delta x \, \alpha_i \tag{3.17}$$

Local Robust Sensitivity Analysis

The defined stepwise methodology and all the variables defined for the global case stay the same for local robust sensitivity analysis. In the local case, however, the decision maker uses local area weights when conducting the sensitivity analysis.

The decision maker has two options to conduct his sensitivity analysis on these local areas. The weight coefficient of elasticity for changing weights can be kept proportional by using Equation 3.1 or assigned directly by decision maker according to the new preferences about the hierarchy weights. The decision maker has also the right to keep some of the weights at their original value assuming that they are unchanging weights for analysis.

Conclusion

This chapter showed that the weighting methodology used for weighting of additive value models affects the correctness of the conducted sensitivity analysis. The new proposed methodology enables the decision maker to conduct the sensitivity analysis with great flexibility using proportionality or parametric assignment of the weight coefficients of elasticity. This brings another dimensions and understandings into the sensitivity analysis. The mathematical notation gives the power to manipulate the

weights in desired areas, allowing some of the weights to stay unchanged if the decision maker is sure of the values of weights in the model.

The following chapter includes an example value hierarchy and represents possible types of sensitivity analysis conducted with the new proposed sensitivity analysis methodology. These possible types of sensitivity analyses are: global and local proportionality exercising the sensitivity analysis in the same fashion the current methodology does. The remaining other two possible types is: global and local parametric sensitivity analysis. In parametric sensitivity analysis, the decision maker can keep some of the weights unchanged, but the weight coefficients of elasticity will be assigned according to the new preferences about the measures' importance other than the proportionality case. However, the proposed parametric analysis has the extra flexibility to keep the dependent changing weights at their original proportionality if it is the desire of the decision maker.

IV. Results

Introduction

This chapter utilizes an example value hierarchy to demonstrate the proposed methodology. All possible sensitivity analysis issues represented in the previous chapter are illustrated and evaluated with this example value hierarchy. The analysis starts with the introduction of the hierarchy and shifts to sensitivity analysis. The following examples are given; global robust proportionality, global robust parametric (according to the decision maker's preference), local robust proportionality and local robust parametric (according to the decision maker's preference) sensitivity analysis. The results are shown and their implications are explained.

Value Hierarchy

An actual deterministic value hierarchy was not selected to avoid issues relating to the specifics of the selected value hierarchy, its weights and possible weight distribution in the sensitivity analysis. Instead, this research uses a notional value hierarchy structure and weights to illustrate the proposed methodology and its implications. The results and applications presented in this chapter are very detailed to allow easy application to other value hierarchies and give the analyst insight about the proposed methodology.

The illustrative value hierarchy is shown on Figure 4-1. It has three sub-objectives and seven measures. Sub-objective1 has two other measures, Sub-objective 2 is described by one measure, and Sub-objective3 has four measures.

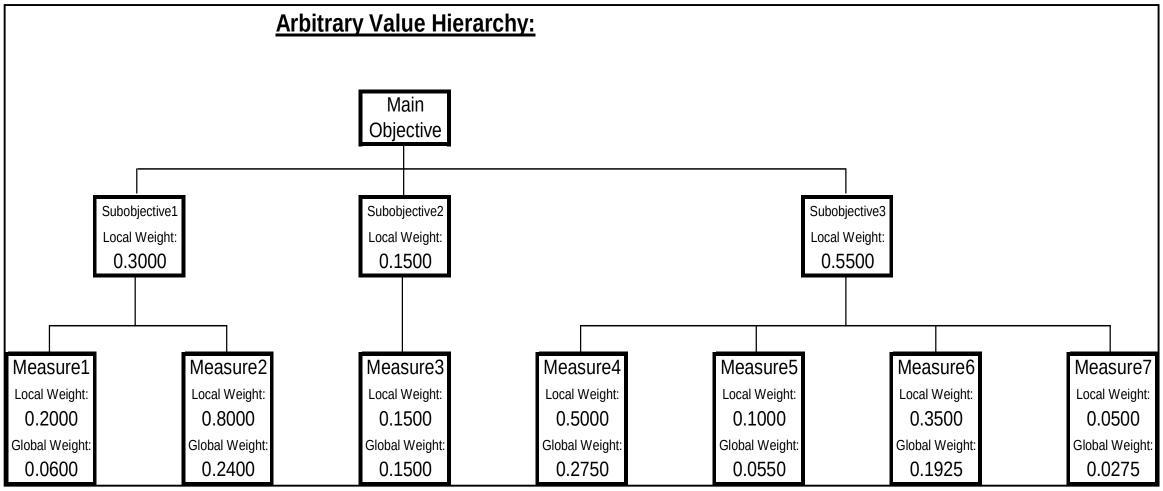

Figure 4-1. The arbitrary value hierarchy evaluated in the research

The weights and single dimensional value function scores are assumed to have been properly elicited from the decision maker. The weighting technique and strategy used is not the focus of the current examples. It is assumed that all the analysis and procedures are applied correctly to build a deterministic value hierarchy. The local and global weights of sub-objectives and measures are given in Figure 4-1.

Scores of Alternatives

The alternatives and single dimensional value function scores are also notionally chosen to support the illustration of the proposed sensitivity analysis approach. The notional scores are shown in Table 4-1. The values were selected to ensure close final scores for alternatives. This helps to demonstrate how sensitivity analysis effects the selection of alternatives. There are five notional alternatives to be analyzed by the decision maker to determine the best decision alternative or policy.

Table 4-1. Notional single dimensional value scores of alternatives

Value Scores:	Att1:	Att2:	Att3:	Att4:	Att5:	Att6:	Att7:
Alternative1:	0.8700	0.5400	0.6500	0.4500	0.7200	0.4300	0.3400
Alternative2:	0.4300	0.6700	0.7700	0.4700	0.5600	0.2600	0.9100
Alternative3:	0.5400	0.4900	0.3500	0.4500	0.6300	0.8500	0.3700
Alternative4:	0.7500	0.8300	0.4600	0.4300	0.3700	0.3400	0.5700
Alternative5:	0.5100	0.9100	0.2800	0.4100	0.1300	0.6000	0.4400

The single dimensional value functions are not considered as a part of this research. The scores provided would be derived from different single dimensional value functions in a real world application of VFT.

The final scores for alternatives are calculated by using Equation 3.9 where the single dimensional value functions scores are multiplied with their associated global weights from value hierarchy and summed together. According to these calculations the final scores and final ranking are shown on Table 4-2.

Table 4-2. Final scores and ranking of arbitrary alternatives

Final Ranking:	
Alternative5:	0.538500
Alternative2:	0.537225
Alternative1:	0.534775
Alternative3:	0.534700
Alternative4:	0.532925

According to the final calculations of the scores Alternative 5 is the highest-ranking alternative and would be selected by the decision maker if selections were based on score alone. The analysis goes further to look at a sensitivity analysis of the weights and see how robust the decision is in terms of changes in the weights of the hierarchy. The sensitivity analysis to be conducted in this research is one-way sensitivity analysis of the

weights. It takes one weight and analyzes the final scores according to the new weight distribution caused by the change in the selected sensitivity weight.

Global Robust Sensitivity Analysis

Global robust sensitivity analysis is exercised at a global area and assumes that the decision maker has weighted the hierarchy globally. It further assumes that the hierarchy is built to simplify the decision situation and its hierarchical construction (the branches and areas) has nothing to do with the weight distribution. The decision maker builds the hierarchy to simplify the problem. After the model is constructed and measures are developed, the decision maker simplifies the hierarchy to a final tier of all the measures. This final global area shows all attributes and their associated global preferences of the decision maker (see Figure 4-2). The construction in the hierarchy above this final area provides structure and allows the analyst to reach the final stage.

The decision maker has two options in this stage to perform sensitivity analysis. The sensitivity analysis can be conducted according to the current methodology, by changing the value of one weight and keeping other weights proportionally, or can be conducted parametrically, assigning the distribution of the weights according to decision maker's preferences. Some of the current weights may be kept unchanged throughout the sensitivity analysis. The proposed methodology is able to conduct both type of weight sensitivity analysis. The following two examples, using the arbitrary value hierarchy, shows the methodology and the application details.

Global Robust (Proportionality).

Step 1: Decide the Sensitivity Area for Analysis

The decision maker conducts this analysis on the entire weight distribution of the final tier. The analysis is presented step by step from beginning to end. The weight distribution is calculated by using Equation 3.7. The respective new weights (w_s, w_i, and w_u) will be calculated with Equations 3.4, 3.5 and 3.6.

$$\sum\left(w_s^0+\alpha_s\Delta x\right)+\sum\left(w_i^0-\alpha_i\Delta x\right)+\sum\left(w_u^0-\alpha_u\Delta x\right)=1$$

The decision maker selects any weight in a global area to conduct the sensitivity analysis. Assume w_4 is selected as the weight to undergo sensitivity analysis (w_s), where w_s^0 is equal 0.275. As a reminder, global weights are used with this analysis, as it is global in nature. There are no unchanging weights (U = {ø}); and all the other weights except the sensitivity weight belong to set I. In this example, there are seven weights in the selected global area. The weight w_1 represents Measure 1's global weight, the w_2 represents Measure 2's global weight, and so forth. Next, the weights for this analysis are defined in their respective sets in Step 2:

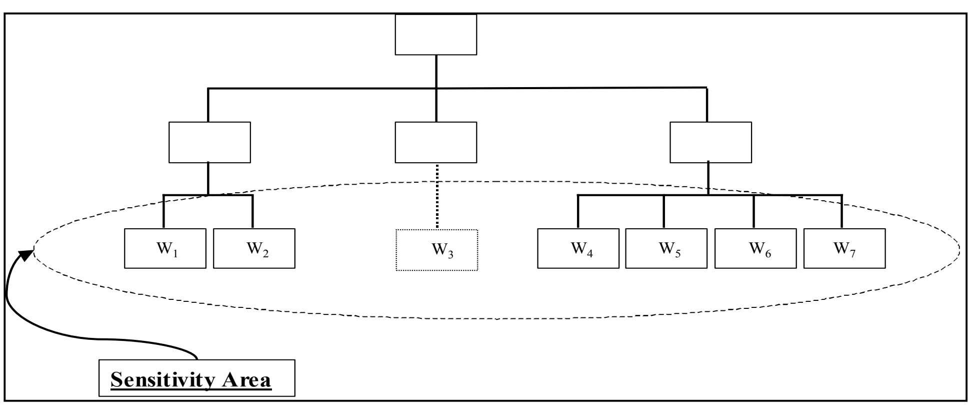

Figure 4-2. The weights and the sensitivity area of interest

Step 2: Define the Sets for the Analysis

$$N = \{w_1, w_2, w_3, w_4, w_5, w_6, w_7\}$$

$$n = |N| = 7$$

$$S = \{w_4\}$$

$$p = |S| = 1$$

$$I = \{w_1, w_2, w_3, w_5, w_6, w_7\}$$

$$r = |I| = 6$$

$$U = \{\ \}$$

$$t = |U| = 0$$

$$n = p + r + t = 7$$

From the hierarchy given in Figure 4-1, it is known that.

$$w_s^0 = w_4^0 = 0.2750$$

w_1^0 =0.0600, w_2^0 =0.2400, w_3^0 =0.1500, w_5^0 =0.0550, w_6^0 =0.1925, and w_7^0 =0.0275

and

$$w_1^0 + w_2^0 + w_3^0 + w_4^0 + w_5^0 + w_6^0 + w_7^0 = 1$$

Step 3: Calculate the Parameters (α_j, bound for Δx)

The weight coefficient of elasticity for sensitivity weight α_4 is 1 by definition. The weight coefficients of elasticity for the dependent weights (elements of set I) are calculated using Equation 3.1 for general robust (proportionality) analysis. The

calculation of parameter α_1 is shown as an example. The values relating to other α parameters are shown on Table 4-3.

$$\alpha_i = \frac{w_i^0}{\sum_{i \in (N-S-U)} w_i^0}$$

$$\alpha_1 = \frac{w_1^0}{w_1^0 + w_2^0 + w_3^0 + w_5^0 + w_6^0 + w_7^0}$$

$$\alpha_1 = \frac{0.0600}{0.0600 + 0.2400 + 0.1500 + 0.0550 + 0.1925 + 0.0275} = 0.082759$$

Table 4-3. The coefficients of elasticity

Elasticity Coefficients:	Value:
α_1	0.082759
α_2	0.331034
α_3	0.206897
α_4	1
α_5	0.075862
α_6	0.265517
α_7	0.037931

The bound for change on sensitivity weight is calculated by using Equation 3.3.

$$-w_s^0 \le \Delta x \le \sum_{i \in I} w_i^0$$

$$\sum_{i \in I} w_i^0 = 0.725$$

To show the validation of Equation 3.2, it is also used to determine the weight coefficients of elasticity.

$$-w_s^0 \leq \Delta x_i \leq \min \frac{w_i^0}{\alpha_i}, \forall i \in I$$

$$\frac{w_1^0}{\alpha_1} = \frac{0.0600}{0.082759} = 0.725$$

$$\frac{w_2^0}{\alpha_2} = \frac{0.2400}{0.331034} = 0.725$$

$$\frac{w_3^0}{\alpha_3} = \frac{0.1500}{0.206897} = 0.725$$

$$\frac{w_5^0}{\alpha_5} = \frac{0.0550}{0.075862} = 0.725$$

$$\frac{w_6^0}{\alpha_6} = \frac{0.1925}{0.265517} = 0.725$$

$$\frac{w_7^0}{\alpha_7} = \frac{0.0275}{0.037931} = 0.725$$

$$-0.2750 \leq \Delta x \leq 0.725$$

The bound for the change in the sensitivity weight is between –0.2750 and 0.725. This allows w_s's global weight to vary from 0 to 1 as expected.

Step 4: Calculate the New Weights According to the Set Parameters

The weights are calculated using Equations 3.4, 3.5, and 3.6 to complete the sensitivity analysis.

Step 5: Calculate the Scores for New Weight Distribution

The new scores are calculated using Equation 3.9. A small sample relating to the scores are shown on Table 4-4.

Table 4-4. Scores of alternatives using global robust (proportionality holds) analysis

Global Weight Att4:		0.000000	0.200000	0.400000	0.600000	0.800000	1.000000
Alternative1:		0.566931	0.543545	0.520159	0.496772	0.473386	0.450000
Alternative2:		0.562724	0.544179	0.525634	0.507090	0.488545	0.470000
Alternative3:		0.566828	0.543462	0.520097	0.496731	0.473366	0.450000
Alternative4:		0.571966	0.543572	0.515179	0.486786	0.458393	0.430000
Alternative5:		0.587241	0.551793	0.516345	0.480897	0.445448	0.410000

Step 6: Show the Results on a Breakeven Chart

A break even chart is presented in Figure 4-3. Alternative 5 is the best choice until the global weight of attribute 4 reaches 0.32. After the global weight of 0.32 for attribute 4, alternative 2 is the prefered choice. All other alternatives are dominated by alternatives 5 and 2 according to the sensitivity analysis. Alternative 1 and Alternative 3 scores overlap, therefore only four alternatives are seen on the chart in Figure 4-3.

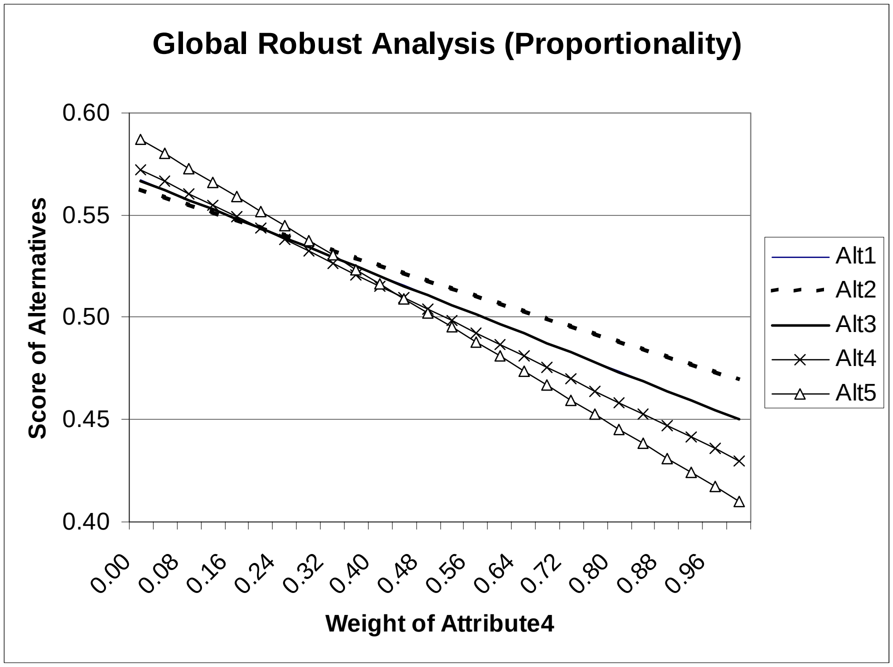

Figure 4-3. Sensitivity analysis results (global proportional)

A more detailed chart fragment showing the changeover is presented on Figure 4-4.

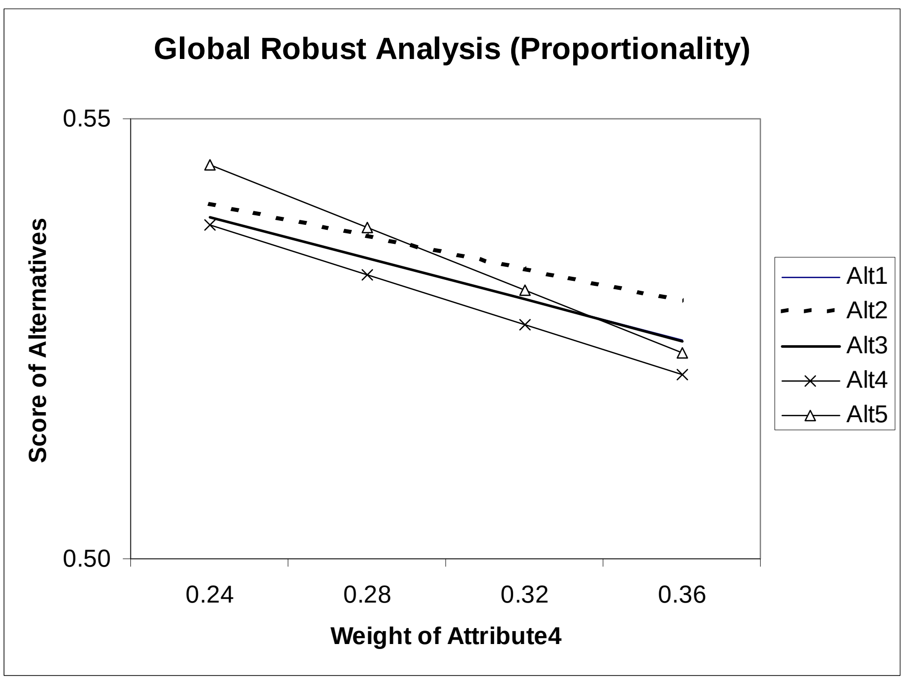

Figure 4-4. Detailed results (global proportional)

Global Robust (Parametric)

Step 1:Decide the Sensitivity Area for Analysis

The decided area is the same that is defined for global proportional case.

Step 2: Define the Sets for the Analysis

Parametric sensitivity analysis differs from proportionality by allowing the decision makers to set the elasticity coefficients of dependent weights (w_i). The set U may no longer be the empty set. The decision makers would fix any elasticity coefficient for any weight in U to 0. However, set U does not have to include any elements. It is assumed in this example that the sensitivity analysis weight is the same, attribute 4's weight (w_4). It is further assumed that the decision maker is sure of the state of four weights (U=$\{w_3, w_5, w_6, w_7\}$). These weights do not change through out the sensitivity analysis (set U). The weight sets according to the new preferences are presented as follow:

$$N = \{w_1, w_2, w_3 w_4, w_5, w_6, w_7\}$$

$$n = |N| = 7$$

$$S = \{w_4\}$$

$$p = |S| = 1$$

$$I = \{w_1, w_2\}$$

$$r = |I| = 2$$

$$U = \{w_3, w_5, w_6 w_7\}$$

$$t = |U| = 4$$

$$n = p + r + t = 7$$

Step 3: Calculate the Parameters (α_i, bound for Δx)

As stated previously, it is assumed the decision maker sets the weight coefficients of elasticity. It is assumed that w_1 and w_2 are set to 0.25 and 0.75 respectively. In this example, they are chosen arbitrarily; however, the relative proportionality with in set I could be maintained. All the coefficients of elasticity are shown on Table 4-5.

Table 4-5. The weight coefficients of elasticity

Elasticity Coefficients:	Value:
α_1	0.25
α_2	0.75
α_3	0
α_4	1
α_5	0
α_6	0
α_7	0

Next, the bound is calculated for Δx by using Equation 3.7.

$$-w_s^0 \le \Delta x_i \le \min \frac{w_i^0}{\alpha_i}, \forall i \in I$$

$$\frac{w_1^0}{\alpha_1} = \frac{0.06}{0.250000} = 0.240$$

$$\frac{w_2^0}{\alpha_2} = \frac{0.24}{0.750000} = 0.320$$

$$-0.2750 \le \Delta x \le 0.240$$

The bound for the change in sensitivity weight is between –0.2750 and 0.240. The global weight of w_s goes from 0 to 0.575. This is expected, the sum of the constant weights is equal to 0.425, therefore the sum of the changing and unchanging weights equals to 1. This is supported through Equation 3.8.

Step 4: Calculate the New Weights According to the Set Parameters

The weights are calculated using Equations 3.4, 3.5, and 3.6 to complete the sensitivity analysis.

Step 5: Calculate the Scores for New Weight Distribution

The new scores are calculated using Equation 3.9. A small sample relating to the scores are shown on Table 4-6.

Table 4-6. Scores of alternatives using global robust (parametric) analysis

Global Weight Att4:		0.000000	0.120000	0.240000	0.320000	0.440000	0.515000
Alternative1:		0.582213	0.561513	0.540813	0.527013	0.506313	0.493375
Alternative2:		0.575725	0.558925	0.542125	0.530925	0.514125	0.503625
Alternative3:		0.549138	0.542838	0.536538	0.532338	0.526038	0.522100
Alternative4:		0.637425	0.591825	0.546225	0.515825	0.470225	0.441725
Alternative5:		0.648500	0.600500	0.552500	0.520500	0.472500	0.442500

Step 6: Show the Results on a Breakeven Chart

The breakeven chart shown on Figure 4-5 shows the best decision choices through out the weight distribution. As seen from the chart, the global manipulation of the weights with parametric preferences for the same sensitivity weight (w_4) gives different results. In this case, alternative 5 is the best decision until the global weight of attribute 4 reaches .30. Then alternative 2 takes over until the global weight for attribute 4 reaches .32. Finally, at this point alternative 3 becomes the best decision for the rest of the weight distribution in this tier.

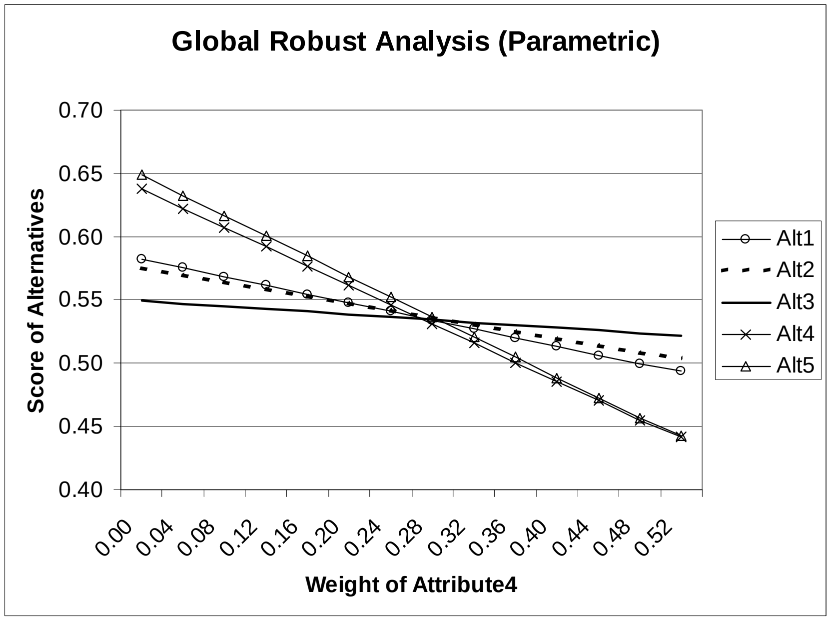

Figure 4-5. Sensitivity analysis results (global parametric)

A more detailed chart fragment showing the changeover is presented on Figure 4-6.

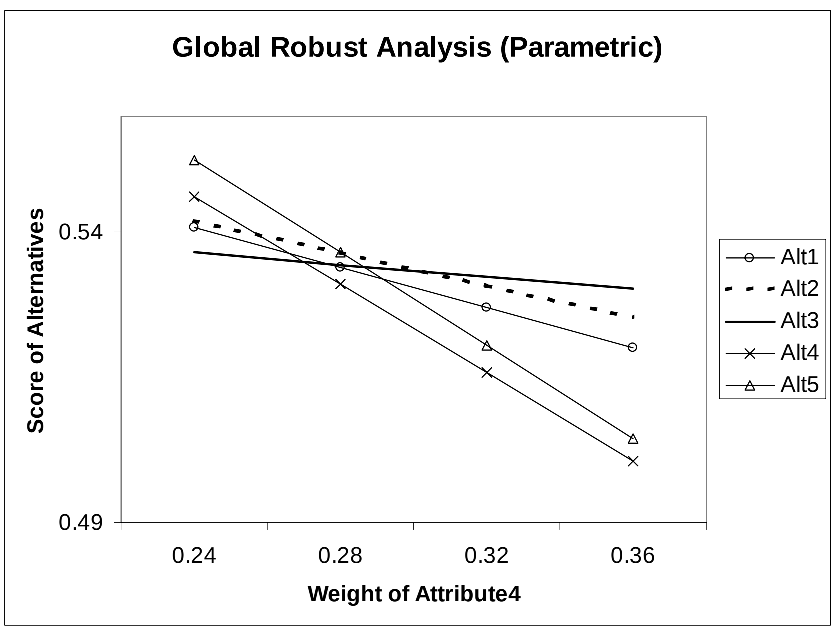

Figure 4-6. Detailed results (global parametric)

Local Robust Sensitivity Analysis

Local robust sensitivity analysis is conducted at local areas and assumes that the decision makers have made their weighting at local areas by using the simplifying features of the value hierarchy. As explained in Chapter 3, the weights in the tiers of different branches of a hierarchy are given independently, by evaluating only the sub-objectives or measures within the tier. The hierarchy is weighted locally using a top to bottom approach. The weights in the tiers of different branches are manipulated independently when the sensitivity analysis is done on a lower tier weight other than the first tier. In the first tier, the weights are both local and global, there is no difference.

This methodology keeps these independent assumptions intact, using the local weights instead of the global weights used during global sensitivity analysis. The weights are manipulated at local area during sensitivity analysis. The decision maker has two options in manipulating of the weights during sensitivity analysis. The decision makers can manipulate the changing weights by keeping the original proportionality between them using Equation 3.1 or they can manipulate them according to their new preferences in the relative importance distribution.

Local Robust (Proportionality)

Step 1: Decide the Sensitivity Area for Analysis

The value hierarchy shown in Figure 4-1 is again used in this example. Assume the analyst decides to exercise sensitivity analysis on the weight belonging to the fourth attribute ($w_s = w_4$). Sensitivity analysis is applied to the area where the attribute four belongs because it is assumed in this part of the research that the decision maker weights the hierarchy locally. The weights and sensitivity area is shown in Figure 4-7.

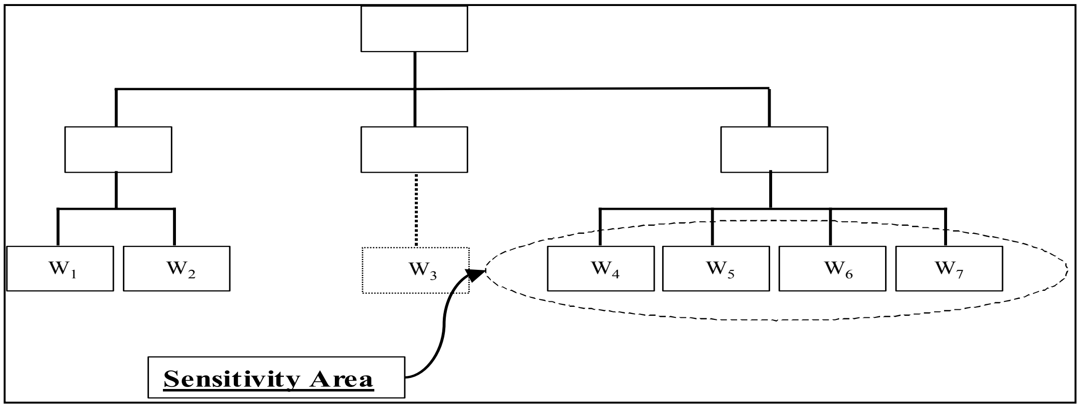

Figure 4-7. The weights and the sensitivity area of interest

Step 2: Define the Sets for the Analysis

The sets in this local analysis are presented as follow:

$$N = \{w_4, w_5, w_6, w_7\}$$

The selected local area has four weights even though the entire hierarchy has seven weights. Weights w_1, w_2, and w_3 are not included in the set N because they are outside the selected local area, or branch of interest.

$$n = |N| = 4$$

$$S = \{w_4\}$$

$$p = |S| = 1$$

$$I = \{w_5, w_6, w_7\}$$

$$r = |I| = 3$$

$$U = \{\ \}$$

$$t = |U| = 0$$

$$n = p + r + t = 4$$

The original local weights are also known from the hierarchy.

$$w_s^0 = w_4^0 = 0.5000$$

$$w_5^0 = 0.1000,\ w_6^0 = 0.3500, \text{ and } w_7^0 = 0.0500$$

$$w_4^0 + w_5^0 + w_6^0 + w_7^0 = 1$$

<u>Step 3: Calculate the Parameters (α_i, bound for Δx)</u>

The weight coefficient of elasticity for sensitivity weight is 1 (α_s). The decision maker keeps the original proportionality between the dependent weights (elements of set I) by using Equation 3.1 in the calculation of their weight coefficients of elasticity. The calculation of parameter α_5 is shown as an example and other weight coefficients of elasticity for other weights are presented in Table 4-7.

$$\alpha_i = \frac{w_i^0}{\sum_{i \in I} w_i^0}$$

$$\alpha_5 = \frac{w_5^0}{w_5^0 + w_6^0 + w_7^0}$$

$$\alpha_5 = \frac{0.1000}{0.1000 + 0.3500 + 0.0500} = 0.2$$

Table 4-7. The weight coefficients of elasticity

Elasticity Coefficients:	Value:
α_4	1.000000
α_5	0.200000
α_6	0.700000
α_7	0.100000

The bound for Δx is calculated by using Equation 3.3.

$$-w_s^0 \le \Delta x \le \sum_{i \in I} w_i^0$$

$$\sum_{i \in I} w_i^0 = 0.500$$

To show the validation of Equation 3.2, this calculation method also is presented.

$$-w_s^0 \le \Delta x_i \le \min \frac{w_i^0}{\alpha_i}, \forall i \in I$$

$$\frac{w_5^0}{\alpha_5} = \frac{0.10}{0.200000} = 0.500$$

$$\frac{w_6^0}{\alpha_6} = \frac{0.35}{0.700000} = 0.500$$

$$\frac{w_7^0}{\alpha_7} = \frac{0.05}{0.100000} = 0.500$$

$$-0.500 \le \Delta x \le 0.500$$

The bound is between –0.500 and 0.500, allowing, the local weight of w_s (w_4) goes from 0 to 1 as is expected.

Step 4: Calculate the New Weights According to the Set Parameters

The weights are calculated using Equations 3.4, 3.5, and 3.6 to complete the sensitivity analysis.

Step 5: Calculate the Scores for New Weight Distribution

The new scores are calculated using Equation 3.1. A small sample relating to the scores are shown on Table 4-8.

Table 4-8. Scores of alternatives using local robust (proportionality holds) analysis

Local Weight Att4:		*0.000000*	*0.218182*	*0.400000*	*0.618182*	*0.800000*	*1.000000*
Alternative1:		0.542750	0.539270	0.536370	0.532890	0.529990	0.526800
Alternative2:		0.513850	0.524050	0.532550	0.542750	0.551250	0.560600
Alternative3:		0.619400	0.582440	0.551640	0.514680	0.483880	0.450000
Alternative4:		0.516150	0.523470	0.529570	0.536890	0.542990	0.549700
Alternative5:		0.560500	0.550900	0.542900	0.533300	0.525300	0.516500

Step 6: Show the Results on a Breakeven Chart

The breakeven chart shown on Figure 4-8 shows the results of this analysis.

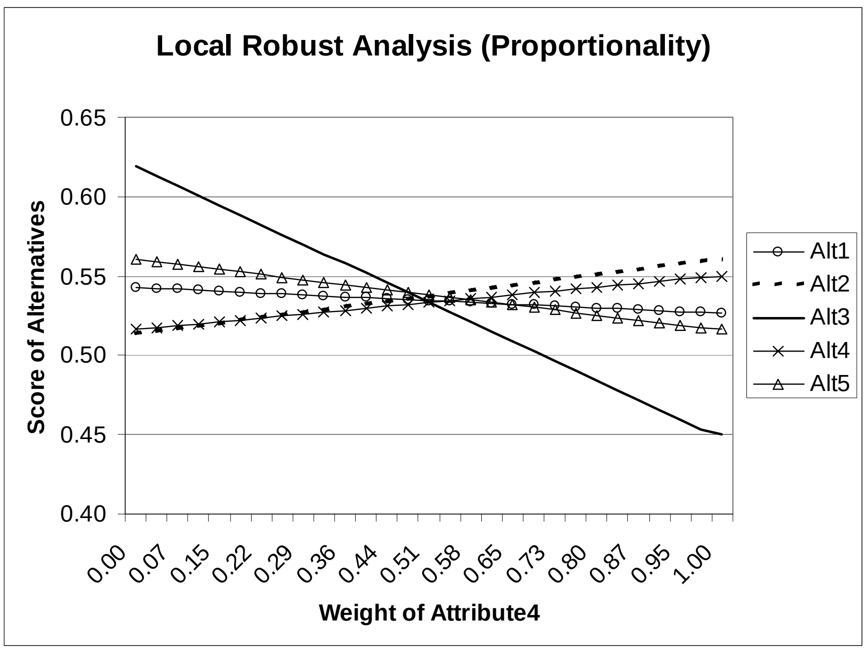

Figure 4-8. Sensitivity analysis results (local proportional)

A more detailed chart fragment showing the changeover is presented on Figure 4-9.

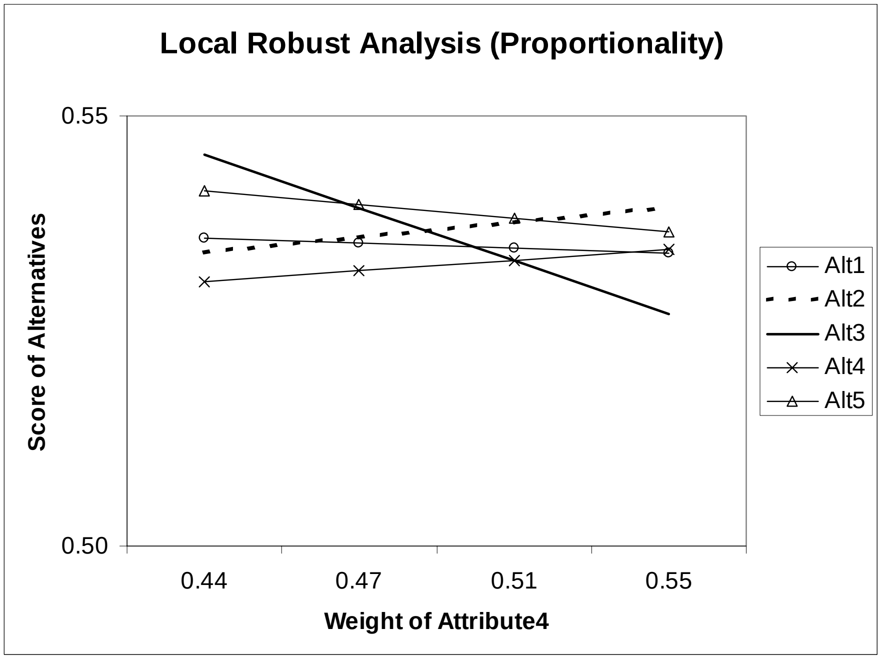

Figure 4-9. Detailed results (local proportional)

As is seen from the chart, the local manipulation of the weights for the same sensitivity weight gives different results than global manipulation of the same weight (w_4). In this case alternative 3 is the best decision until the local weight of attribute 4 reaches .4727. Alternative 5 takes over until the local weight for attribute 4 reaches .5454, and finally alternative 2 is the best decision for the rest of the weight distribution in this tier.

<u>Local Robust (Parametric)</u>

Local robust parametric sensitivity analysis is similar to local robust proportionality case except the parts relating to the weight sets and weight coefficients of elasticity.

Step 1:Decide the Sensitivity Area for Analysis

In this case, the analyst can leave some of the weights unchanged while manipulating the dependent (changing weights) according to the decision makers' new preferences. It is assumed in this part of the analysis that the decision maker wants to keep the weight of attributes5 constant and he also wants the weight coefficients of elasticity for attribute6 and attribute7 to maintain their original proportionality. This allows the decision maker to keep relative proportionality within set I. The sensitivity area is the same as it is in local proportional case.

Step 2: Define the Sets for the Analysis

The weight sets according to the new preferences are presented as follow:

$$N = \{w_4, w_5, w_6, w_7\}$$

$$n = |N| = 4$$

$$S = \{w_4\}$$

$$p = |S| = 1$$

$$I = \{w_6, w_7\}$$

$$r = |I| = 2$$

$$U = \{w_5\}$$

$$t = |U| = 1$$

$$n = p + r + t = 4$$

The original local weights are also known from the hierarchy.

$$w_s^0 = w_4^0 = 0.5000$$

$$w_5^0 = 0.1000,\ w_6^0 = 0.3500, \text{ and } w_7^0 = 0.0500$$

$$\overset{0}{w_4} + \overset{0}{w_5} + \overset{0}{w_6} + \overset{0}{w_7} = 1$$

Step 3: Calculate the Parameters (α_i, bound for Δx)

The weight coefficient of elasticity for sensitivity weight is 1 (α_s). The decision maker keeps the original proportionality between the dependant weights (elements of set I) by using Equation 3.1 in the calculation of their weight coefficients of elasticity. The calculation of parameter α_6 is shown as an example and other weight coefficients of elasticity for other weights are presented in Table 4-9.

$$\alpha_i = \frac{\overset{0}{w_i}}{\sum_{i \in I} \overset{0}{w_i}}$$

$$\alpha_6 = \frac{\overset{0}{w_6}}{\overset{0}{w_6} + \overset{0}{w_7}}$$

$$\alpha_6 = \frac{0.3500}{0.3500 + 0.0500} = 0.875$$

Table 4-9. The weight coefficients of elasticity

Elasticity Coefficients:	Value:
α_4	1
α_5	0
α_6	0.875
α_7	0.125

The bound for Δx is calculated by using formula 3.2

$$-w_s^0 \le \Delta x_i \le \min \frac{w_i^0}{\alpha_i}, \forall i \in I$$

$$\frac{w_6^0}{\alpha_6} = \frac{0.35}{0.875000} = 0.400$$

$$\frac{w_7^0}{\alpha_7} = \frac{0.05}{0.125000} = 0.400$$

$$-0.500 \le \Delta x \le 0.400$$

Even if a parametric analysis is done, the bound for Δx can be also calculated using Equation 3.3 because the original proportionality between changing weights are kept at their original value.

$$-w_s^0 \le \Delta x \le \sum_{i \in I} w_i^0$$

$$\sum_{i \in I} w_i^0 = 0.400$$

Step 4: Calculate the New Weights According to the Set Parameters

After the bound is decided the new weights in the tier are calculated with the aid of Equations 3.4, 3.5, and 3.6.

Step 5: Calculate the Scores for New Weight Distribution

The new scores of alternatives are calculated with the Equation 3.9. A sample of final scores is shown on Table 4-10.

Table 4-10. Scores of alternatives using local robust (parametric) analysis

Local Weight Att4:		*0.000000*	*0.327273*	*0.472727*	*0.618182*	*0.763636*	*0.900000*
Alternative1:		0.526181	0.531806	0.534306	0.536806	0.539306	0.541650
Alternative2:		0.501819	0.524994	0.535294	0.545594	0.555894	0.565550
Alternative3:		0.628200	0.567000	0.539800	0.512600	0.485400	0.459900
Alternative4:		0.516081	0.527106	0.532006	0.536906	0.541806	0.546400
Alternative5:		0.585250	0.554650	0.541050	0.527450	0.513850	0.501100

Step 6: Show the Results on a Breakeven Chart

The breakeven chart on Figure 4-10 shows the best decision alternatives throughout the weight distribution pattern.

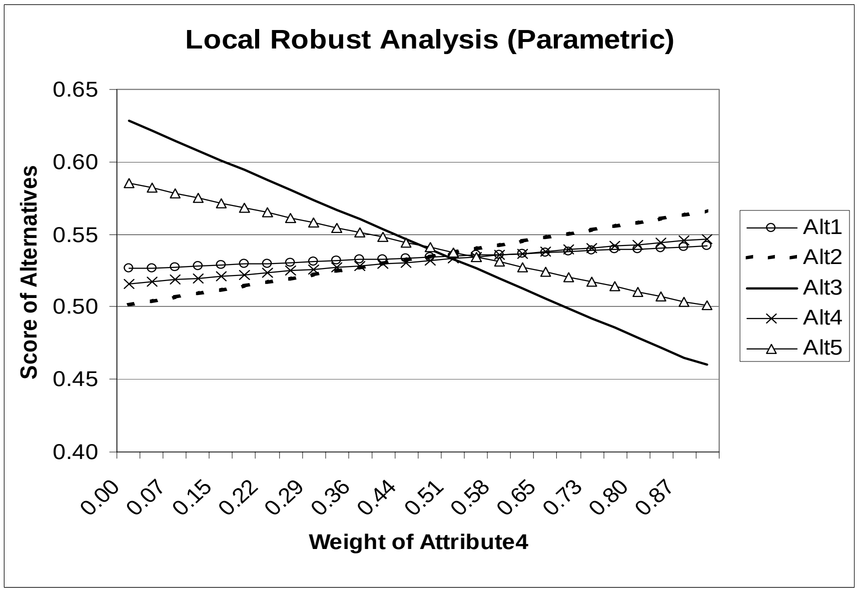

Figure 4-10. Sensitivity analysis results (local parametric)

A more detailed chart fragment showing the changeover is presented on Figure 4-11.

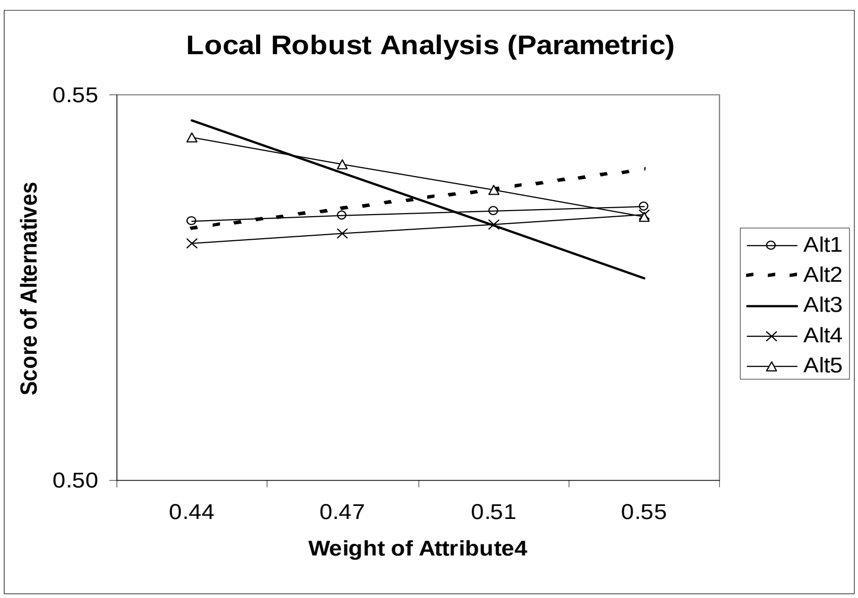

Figure 4-11. Detailed results (local parametric)

As it seen from the chart, the local parametric manipulation of the weights gives the same general results as local robust proportionality manipulation the same weights. The break-even points are different than the previous analysis. In this case, alternative 3 is the best decision until the local weight of attribute 4 reaches .4727. Alternative 5 takes over until the local weight for attribute 4 reaches .5090, and finally alternative 2 is the best decision for the rest of the weight distribution in this tier.

Summary

This chapter includes all possible sensitivity analysis examples applicable to the proposed sensitivity analysis methodology as currently defined. It presents the application of area selection and shows all the calculations numerically relating to this selection. Additional ideas about the interpretation of breakeven charts are given along with pictorial representations of the conducted sensitivity analysis.

V. Conclusion

Introduction

This chapter summarizes the proposed sensitivity analysis methodology for hierarchical additive value models and its implications. Furthermore, the limitations of the methodology are identified and possible advancement areas for future research are given.

Conclusions

Hierarchical value models are a way to represent additive weighted sum models systematically. Hierarchical structures in decision analysis help decision makers, and analysts to simplify the problem by breaking it into more comprehensive parts and to reach the conclusion quickly and more effectively. The weighting strategy, used during their evaluation process, can cause difficulties during sensitivity analysis process. The sensitivity analysis should be conducted according to the weighting strategy; the sensitivity analysis should be conducted locally if the hierarchy is weighted locally or globally if the hierarchy is weighted globally. Therefore, the analyst conducting the sensitivity analysis must take the weighting strategy into consideration. The current methodology in the literature does not provide a consistent mathematical representation for capturing sensitivity analysis. Predominantly, sensitivity analysis techniques within the literature and software complete the sensitivity analysis globally. Global sensitivity analysis may not reflect the exact preferences of the decision makers, especially when the hierarchy is locally weighted. This type of analysis can cause the decision makers to select the wrong alternative or to make incorrect decisions.

The proposed methodology accounts for local or global manipulation of weights. Additional flexibility is provided to the analyst by allowing the decision makers to conduct their analysis parametrically. The decision makers can implement new preferences during the sensitivity analysis other than the preferences assigned during the structuring phase, which would be proportionality, of the value hierarchy. Some of the weights may be held constant or unchanged while performing sensitivity analysis.

The proposed methodology provides a common mathematical framework for sensitivity analysis of hierarchical additive value models and standardizes the sensitivity analysis notation and terminology. Finally, the proposed method gives flexibility to the analyst and decision makers through the use of parametric sensitivity analysis.

Limitations

The proposed methodology does not have any limitations when it is implemented according to the weighting strategy used. However, the analysts cannot do global parametric sensitivity analysis to a locally weighted value hierarchy. This is due to the constraint of local weights summing to 1 within each branch of the hierarchy. If the hierarchy is weighted locally and the analysis done globally, this would destroy the independent weighted structure of the branches. This methodology is not able to decide the proportional distribution of the weights in different branches in a locally weighted hierarchy. This methodology is designed to handle the weights within a branch of the same tier if it is weighted locally.

Recommendations and Future Research

The proposed methodology asks the decision makers for preference distributions about the weights when it is implemented parametrically. The method of determining the weight coefficients of elasticity should be evaluated more closely. A possible avenue for research involves multiple decision makers and focusing on the extraction methods of the weight coefficients of elasticity.

Future research can also focus on global parametric application of the proposed methodology on locally weighted additive hierarchical value models. This would addresses the discussed limitations of the proposed methodology by adding additional constraints to the mathematical formulation.

Summary

The proposed robust sensitivity analysis methodology handles the sensitivity analysis problems of weighted additive hierarchical value models with great flexibility. Its application provides the decision analysis community with a common approach to handle sensitivity analysis. This research fills a current void in the literature of hierarchical weighted sum models.

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Vita

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14. ABSTRACT
There is a wide array of multi-attribute decision analysis methods and associated sensitivity analysis procedures in the literature. However, there is no detailed discussion of sensitivity analysis methods solely relating to additive hierarchical value models. The currently available methodology in the literature is unsophisticated and can be hard to implement into complex models. The methodology proposed in this research builds mathematical foundations for a robust sensitivity analysis approach and extends the current methodology to a more powerful form. The new methodology is easy to implement into complex hierarchical value models and gives flexible and dynamic capabilities to decision makers during sensitivity analysis. The mathematical notation is provided in this study along with applied examples to demonstrate this methodology. Global and local sensitivity analysis are considered and implemented using the proposed robust technique. This research provides consistency and a common standard for the decision analysis community for sensitivity analysis of multi-attribute deterministic hierarchical value models.

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